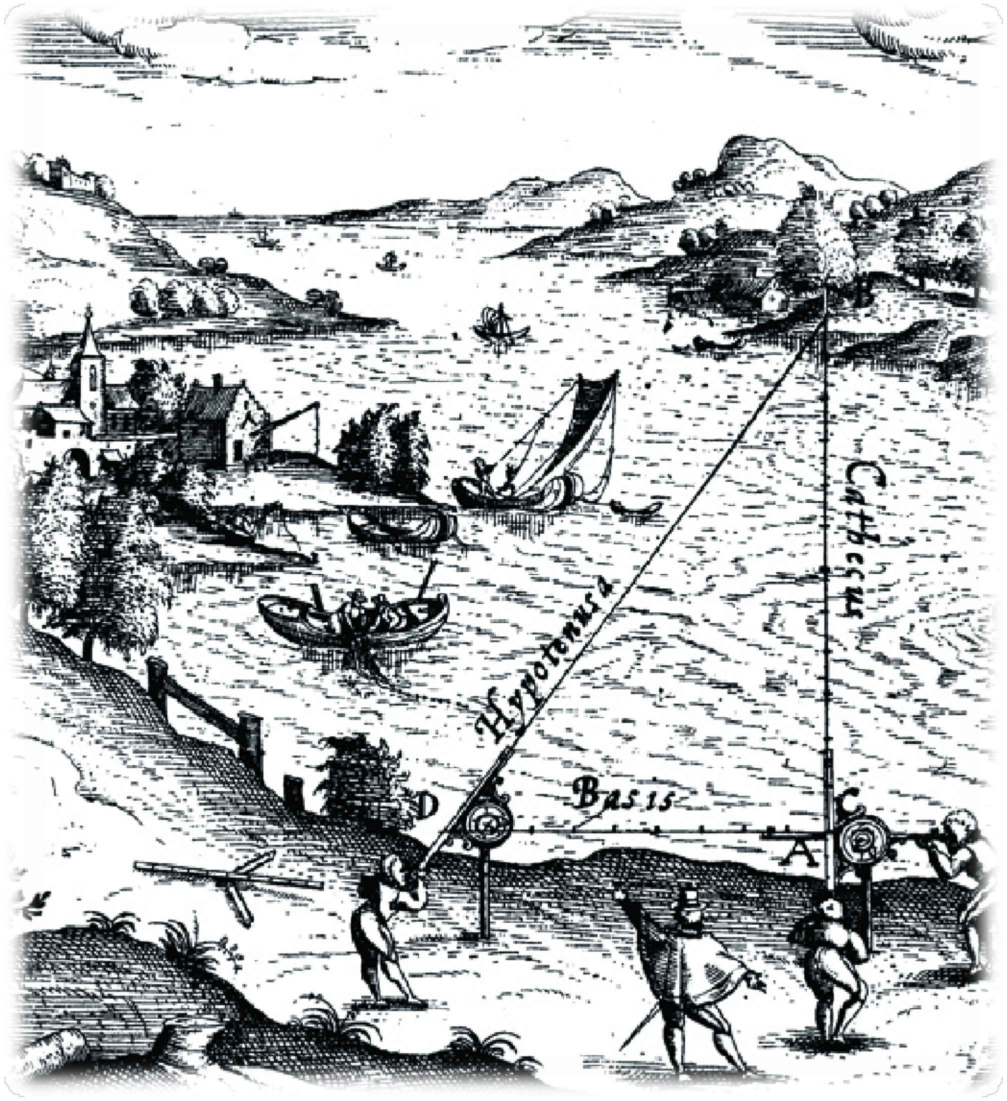
CHAPTER 8
Trilateration and Triangulation(3 hr)



Cathecus
Hypotenusa
Basis
D
C
A

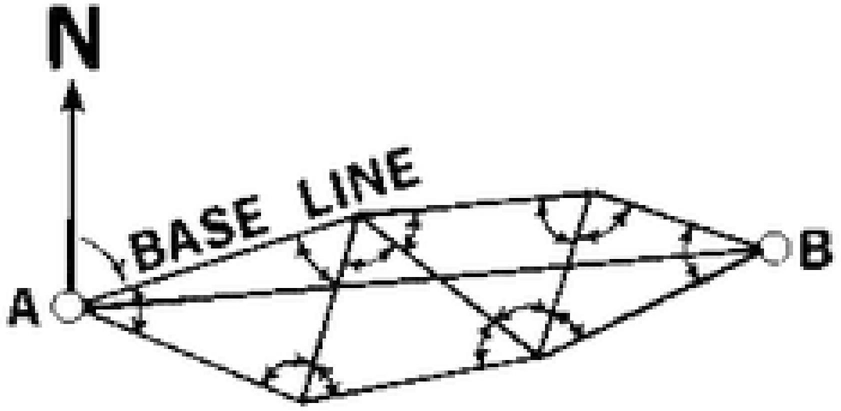
Chp 8. Trilateration and Triangulation (3 Hour)

- Principles of Trilateration
- Principles and Classification of Triangulation Systems (pu2013)
- Strength of Figure
- Satellite Stations and Inter – Visibility of Triangulation Stations (pu,2011)
- Instruction on Field Works

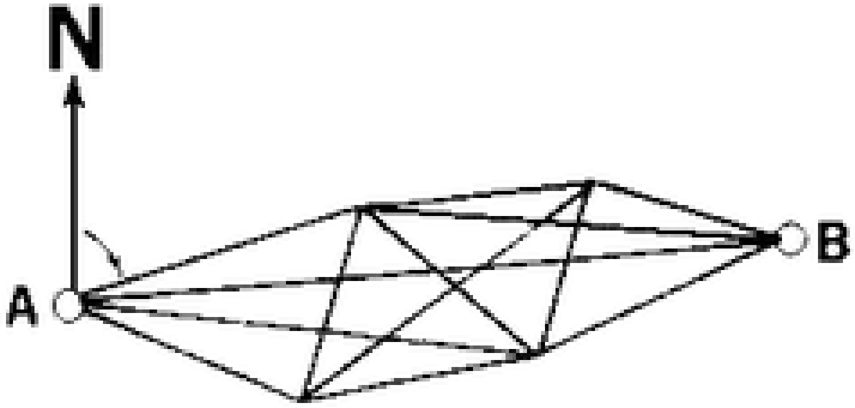
D Short note – traingulation and trilateration (Pu 2008,09*2,010,012)

D Distinguish between traingulation and trilateration (2011)

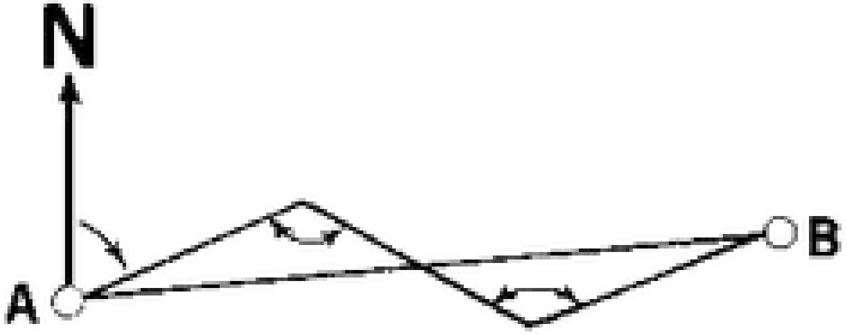
D Advantages of Trilateration (2011)


N
BASE LINE
A
B

TRIANGULATION



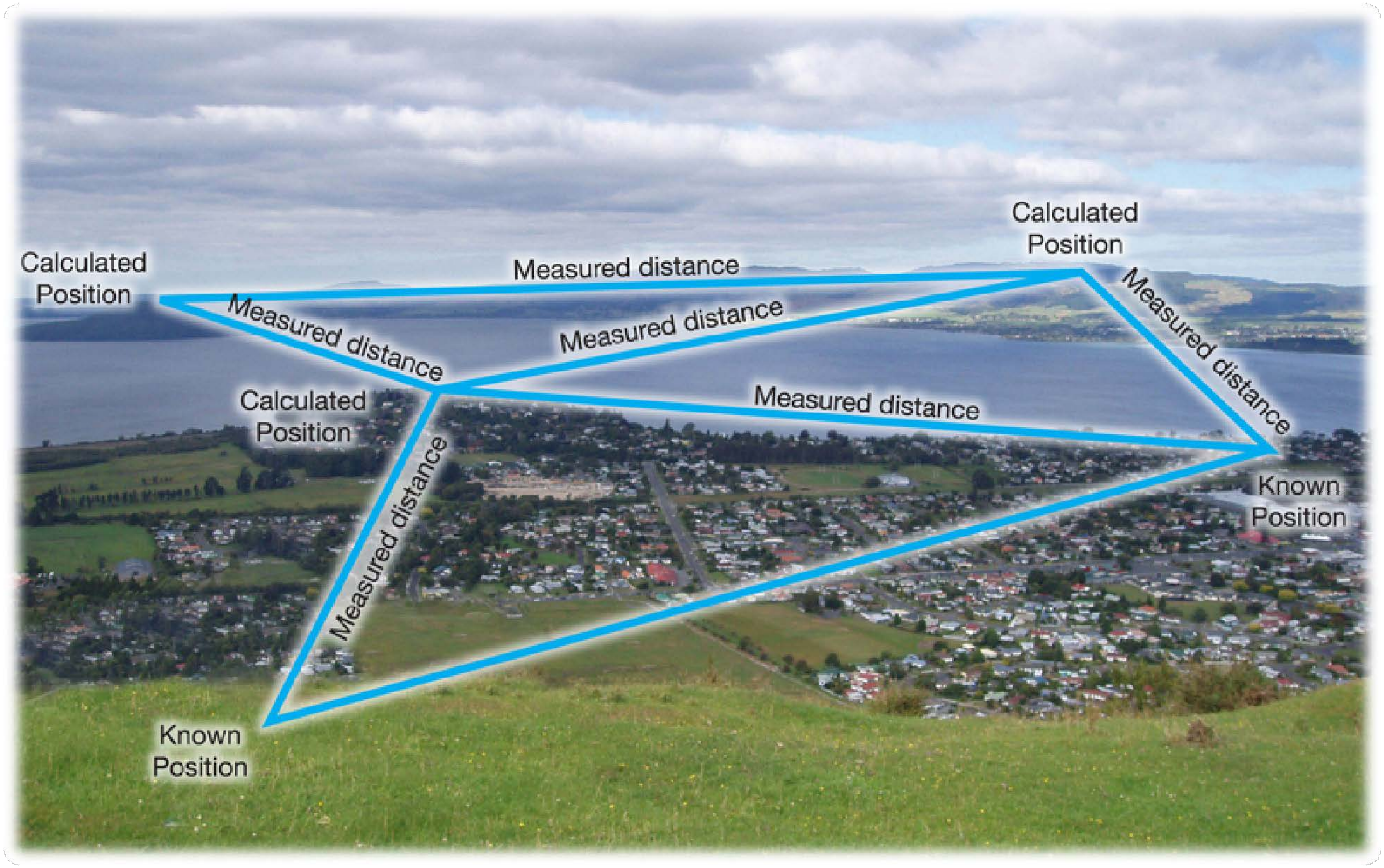
TRILATERATION



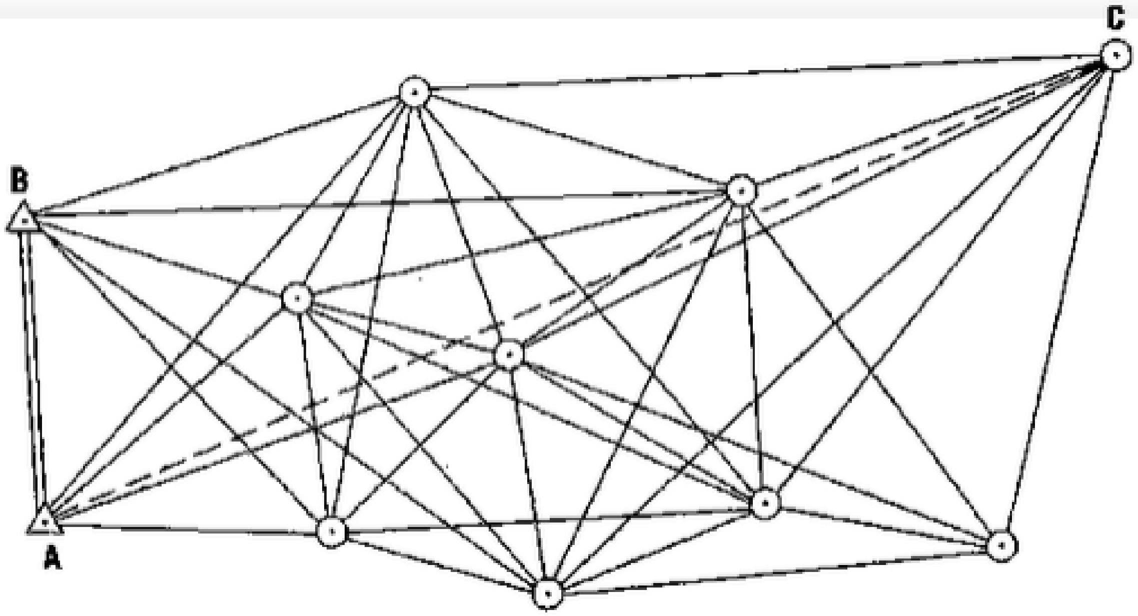
TRAVERSE

Trilateration

- A surveying method in which the lengths of all sides of a chain of triangles, polygons, or quadrilaterals (or any combination of them) aremeasured with an electronic instrument or others; the angles then may be computed from these field measurements.
- Uses in the construction of a chain or network of interconnected triangles ina given area and the measurement of all three sides of each triangle.
- Angles of the triangles and the coordinates of their vertices are determined by trigonometric computations.
- In contrast to triangulation, it does not involve the measurement of angles.
- Trilateration has the same purpose as triangulation.


Calculated Position
Calculated Position
Measured distance
Measured distance
Measured distance
Measured distance
Calculated Position
Measured distance
Measured distance
Known Position
Measured distance
Known Position

Trilateration Network

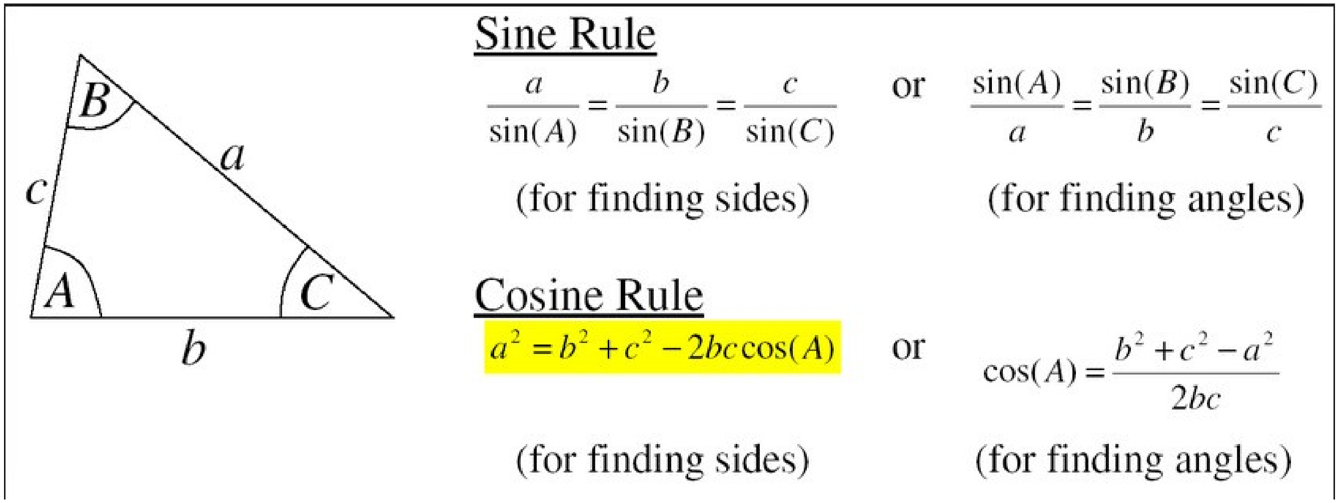


KNOWN DATA:
- Length of base line AB.
- Latitude and longitude of points A and B.
- Azimuth of line AB.

MEASURED DATA:
- Length of all triangle sides.

COMPUTED DATA:
- Latitude and longitude of point C, and other new points.
- Length and azimuth of line AC.
- Length and azimuth between any two points.

A TRILATERATION NET



Sine Rule

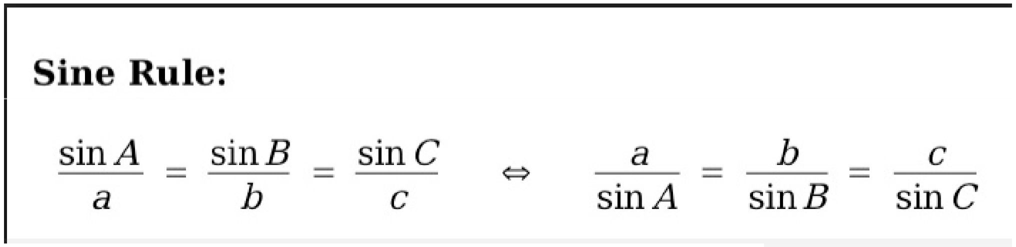
$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)} \quad \text{or} \quad \frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$

(for finding sides) (for finding angles)

Cosine Rule

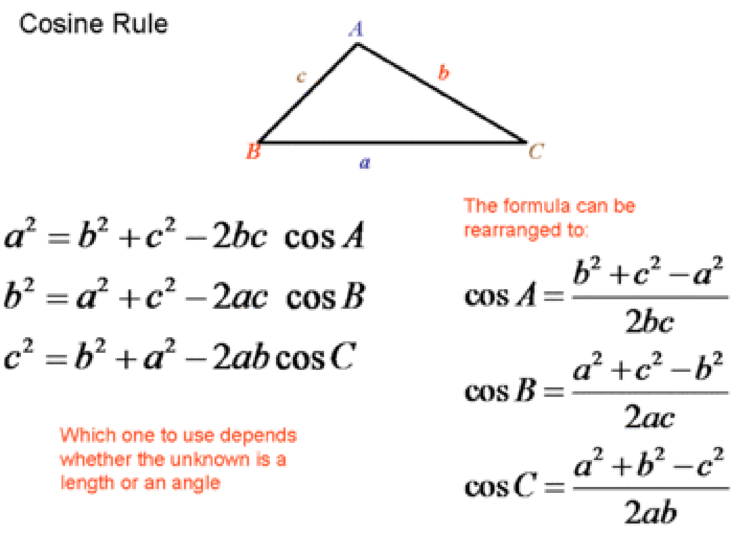
$$a^2 = b^2 + c^2 - 2bc\cos(A) \quad \text{or} \quad \cos(A) = \frac{b^2 + c^2 - a^2}{2bc}$$

(for finding sides) (for finding angles)



Sine Rule:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \Leftrightarrow \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



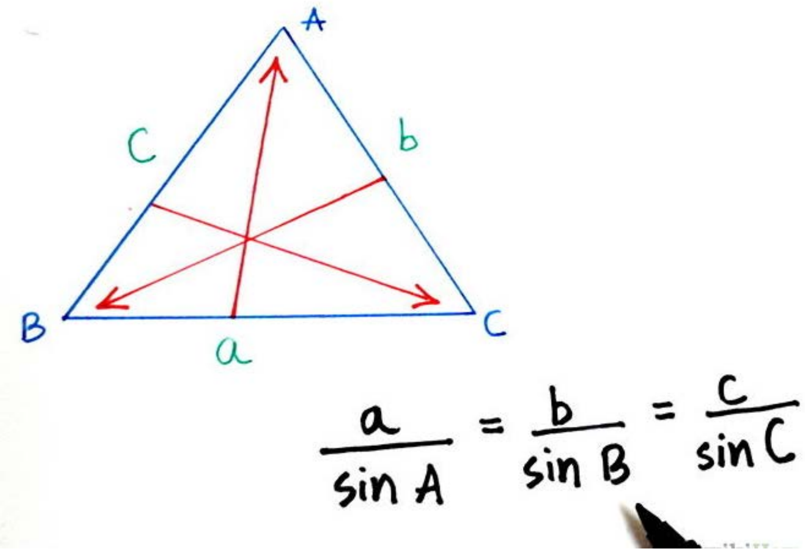
Cosine Rule

$$a^2 = b^2 + c^2 - 2bc\ \cos A$$
$$b^2 = a^2 + c^2 - 2ac\ \cos B$$
$$c^2 = b^2 + a^2 - 2ab\cos C$$

Which one to use depends whether the unknown is a length or an angle

The formula can be rearranged to:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$
$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$
$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Sine Rule & cosine rule (for both methods)

Trilateration and its Principles

- Trilateration is a highly accurate and precise method of establishing and expanding horizontal control.
- Method of control survey in which a network of triangles is used as in triangulation system.
- All the three sides of each triangle are measured in the field with the distance measuring instruments(EDMs, tapes, other apparatus).
- Horizontal angles are not measured in the field.
- Angles in a trilateration system are computed indirectly from the lengths of the sides of triangle by cosine formula.
- Few horizontal angles are also sometimes measured to provide a check on computed angles.
- Trilateration is adjusted after the computation of the angles and then coordinates of the stations are determined.
- Astronomical observations for azimuth are also made at selected stations.
- Vertical angles are also measured where elevations have not been established.

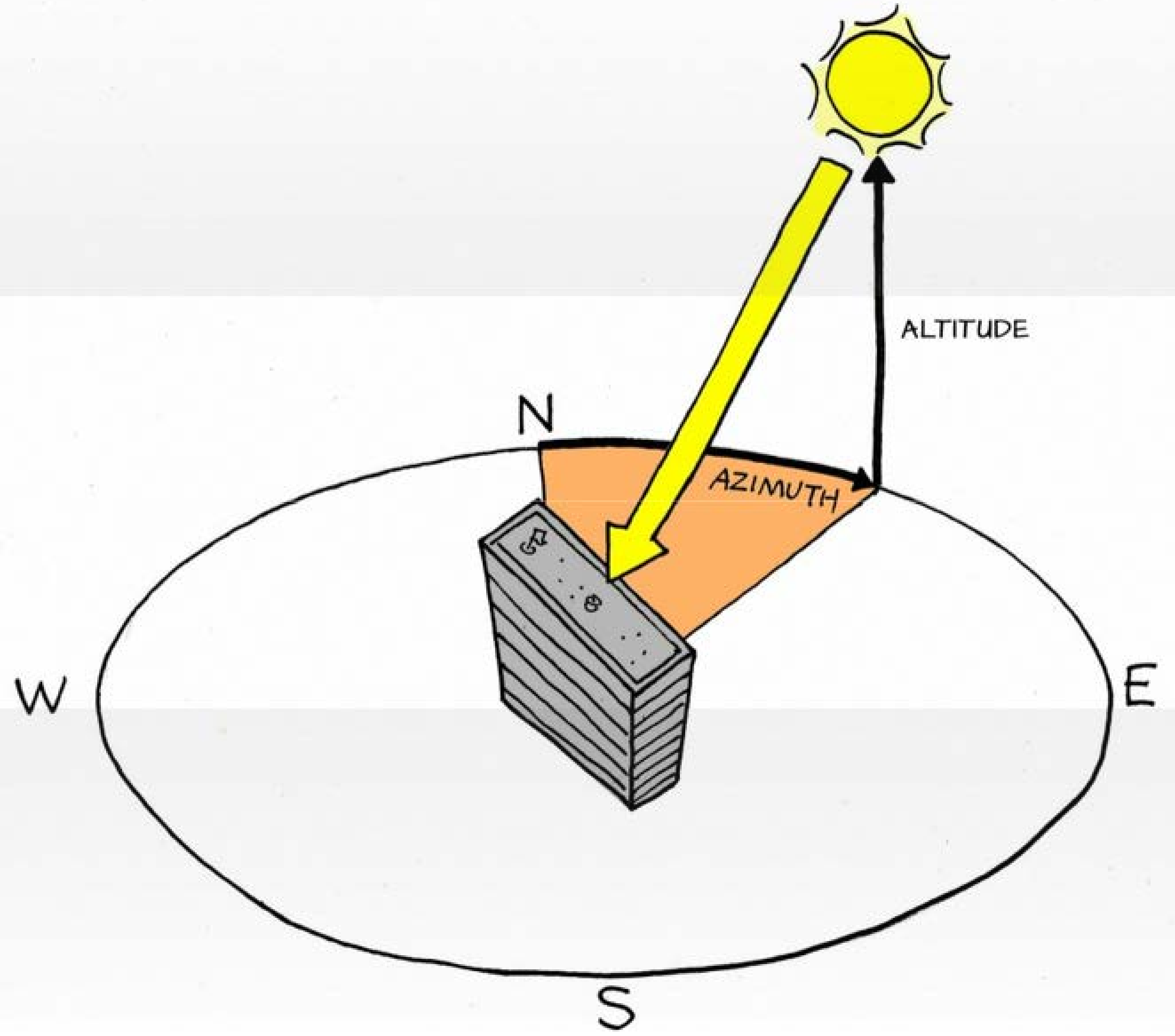
Cosine Rule:

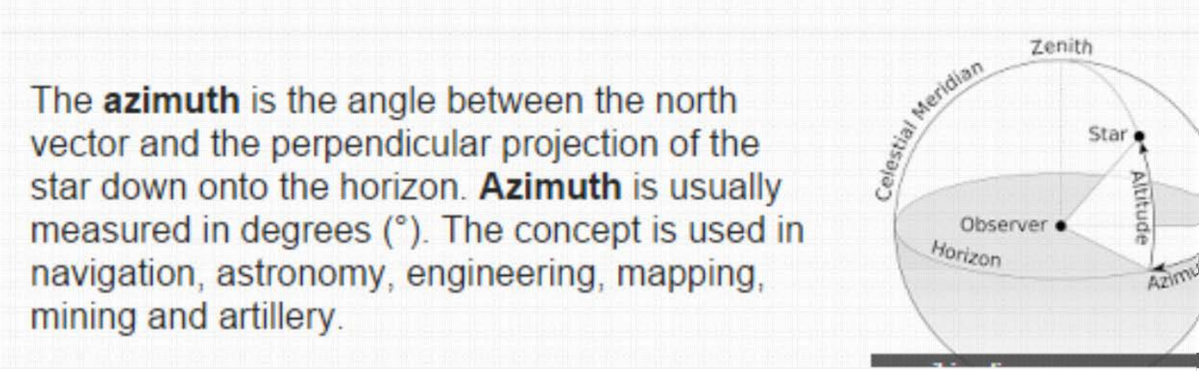
$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$2bc\cos A = b^2 + c^2 - a^2$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

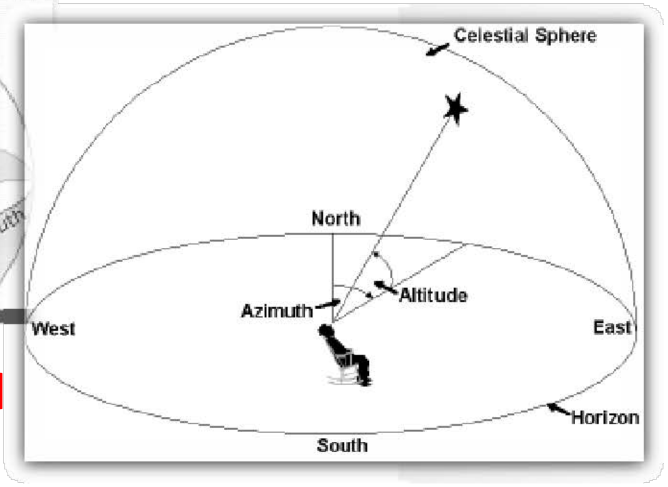
$$A = \cos^{-1}\left(\frac{b^2 + c^2 - a^2}{2bc}\right)$$


ALTITUDE
N
AZIMUTH
W
E
S

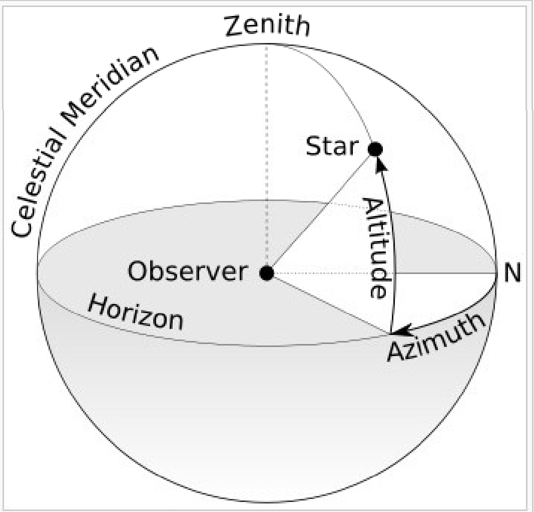


The **azimuth** is the angle between the north vector and the perpendicular projection of the star down onto the horizon. **Azimuth** is usually measured in degrees (°). The concept is used in navigation, astronomy, engineering, mapping, mining and artillery.

Zenith
Celestial Meridian
Star
Altitude
Observer
Horizon
Azimuth


Celestial Sphere
North
Altitude
Azimuth
West
East
Horizon
South

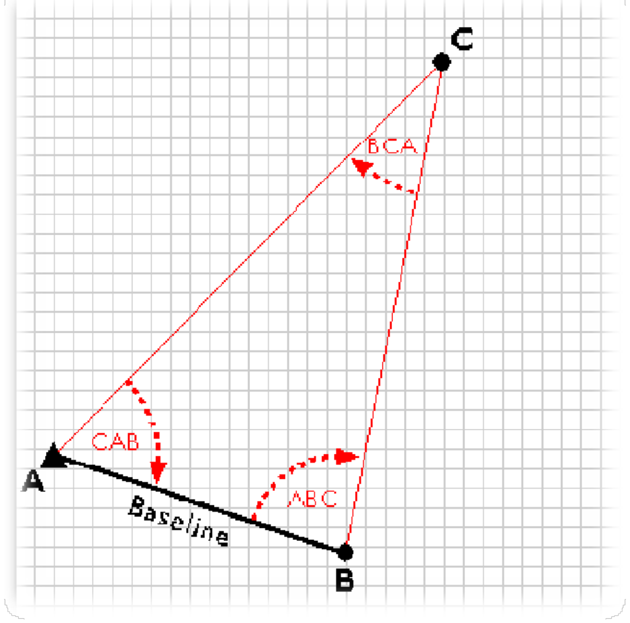
- An azimuth is an angular measurement in a spherical coordinate system.
- An example is the position of a star in the sky. The star is the point of interest, the reference plane is the horizon or the surface of the sea, and the reference vector points north.
- The azimuth is the angle between the north vector and the perpendicular projection of the star down onto the horizon.
- Azimuth is usually measured in degrees (°).
- The concept is used in navigation, astronomy, engineering, mapping, mining.

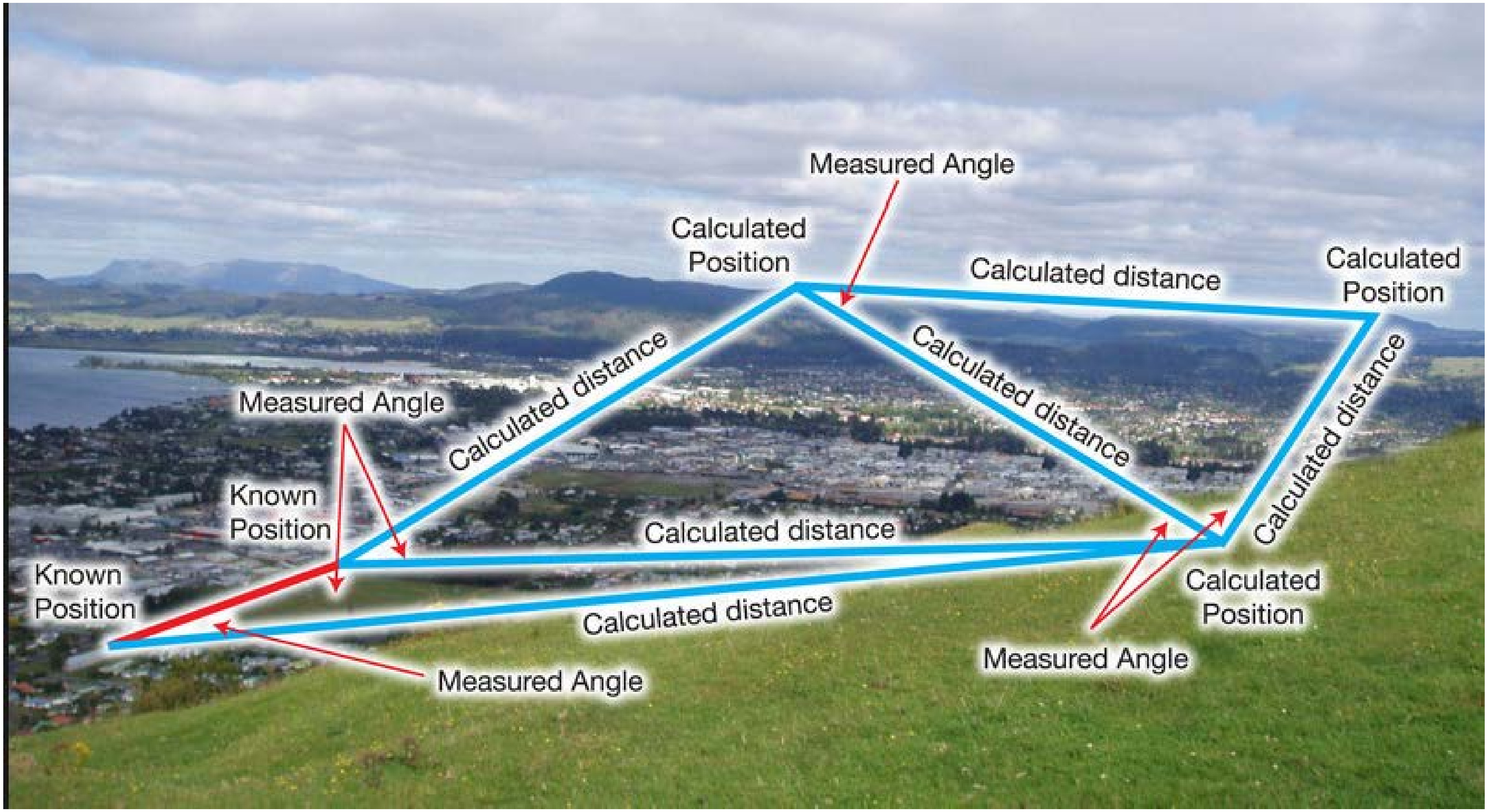

Zenith
Celestial Meridian
Star
Altitude
Observer
N
Horizon
Azimuth

The azimuth is the angle formed between a reference direction (North) and a line from the observer to a point of interest projected on the same plane as the reference direction

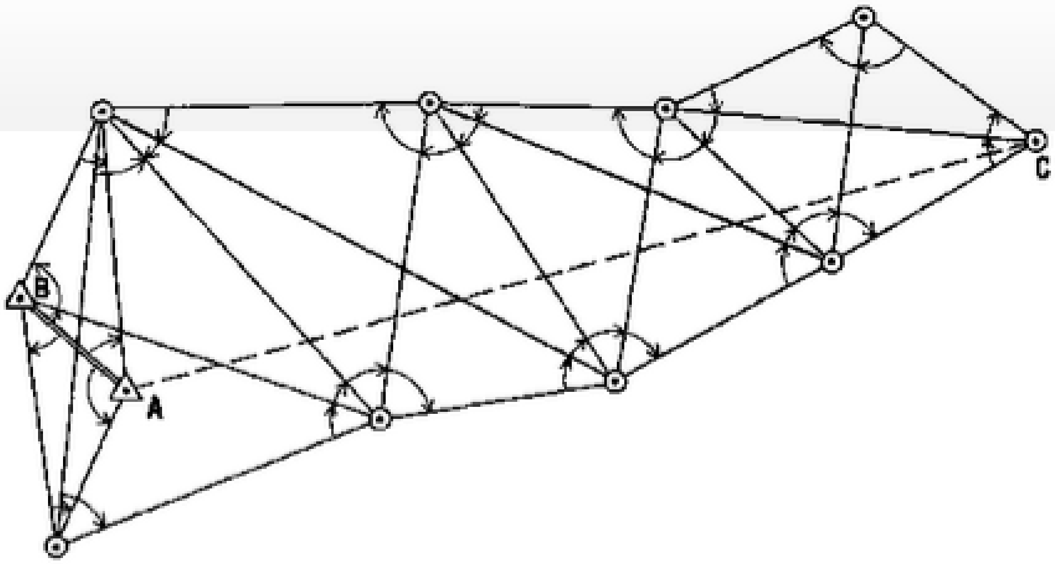
Triangulation and its Principles

- It is the process of measuring the angles of a chain or network of triangles formed by stations marked on the surface of the earth.
- The system consists of a number of interconnected triangles in which the length of only one base line and the angles of the triangles are measured very precisely which are used to calculate the coordinate of vertices.




Measured Angle
Calculated Position
Calculated distance
Calculated Position
Calculated distance
Calculated distance
Measured Angle
Calculated distance
Known Position
Calculated distance
Known Position
Calculated Position
Calculated distance
Measured Angle
Measured Angle

Note : Red – base line



KNOWN DATA:
- Length of base line AB.
- Latitude and longitude of points A and B.
- Azimuth of line AB.

MEASURED DATA:
- Angles to new control points.

COMPUTED DATA:
- Latitude and longitude of point C, and other new points.
- Length and azimuth of line AC.
- Length and azimuth of all other lines.

A SIMPLE TRIANGULATION NET

Er. Pramesh Hada, Assistant Professor, nec

Principle of triangulation

- If all the three angles and the length of one side of a triangle are known, then by trigonometry the lengths of the remaining sides of the triangle can be calculated.
- Again, if the coordinates of any vertex of the triangle and azimuth of any side are also known, then coordinates of the remaining vertices may be computed.

Triangulation

Background

- In survey after the establishment of monuments it is necessary to determine the ground position i.e. coordinates of those monument station which prevent the accumulation of errors and will form a frame work on which entire survey is to be based. This is called control point establishment.
- Such provision of control point can be made either one or combination of both the following methods

1. Traverse
2. Triangulation

   Triangulation is considered to be more accurate than traversing as there is less accumulation of errors than that in traverse.

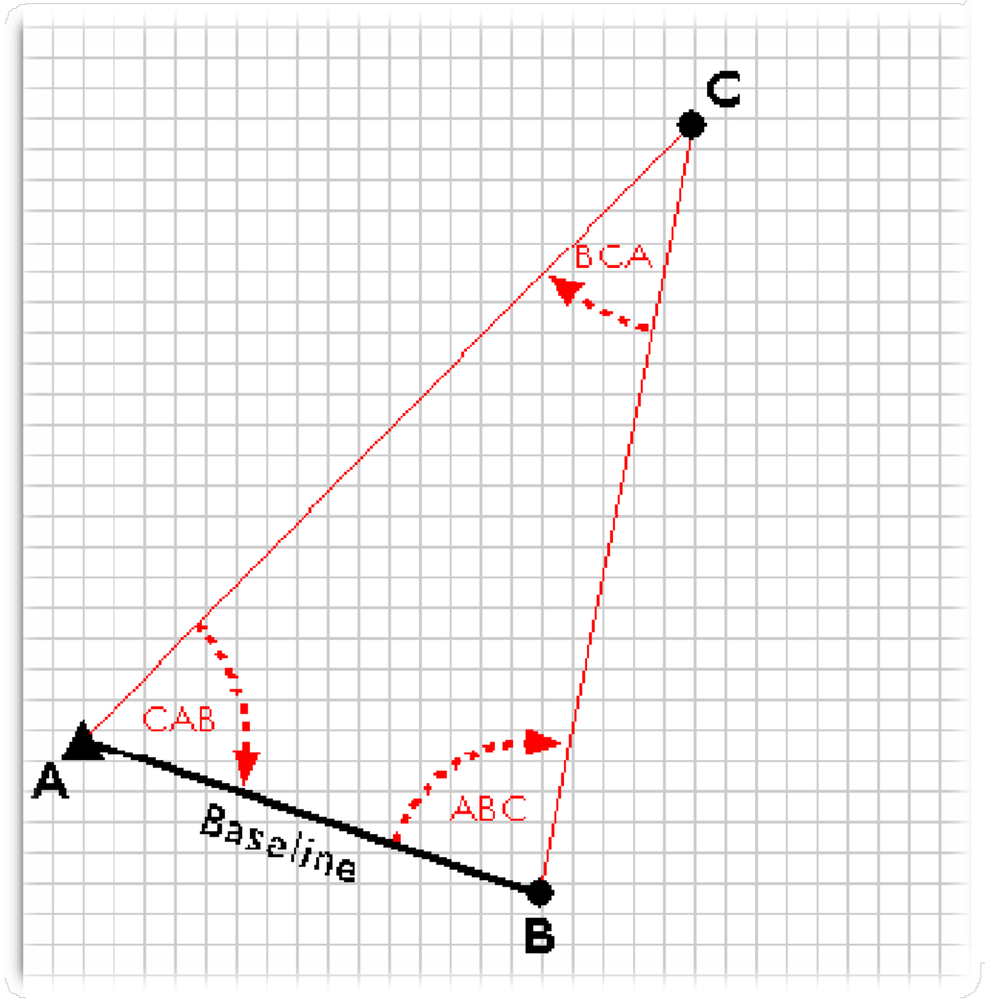
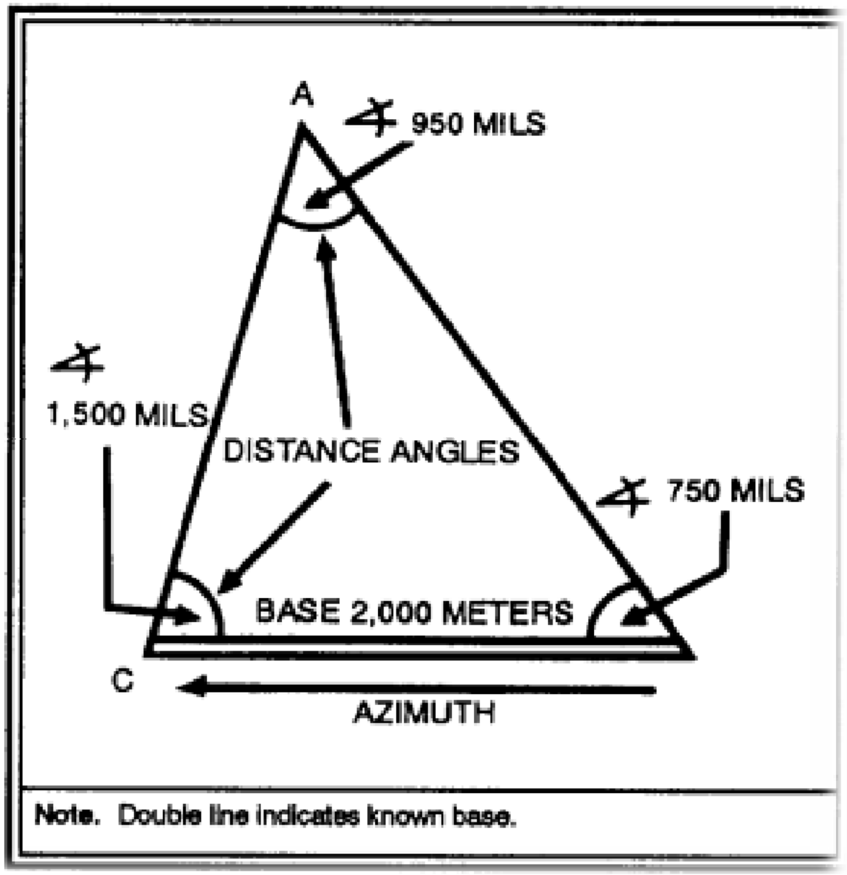
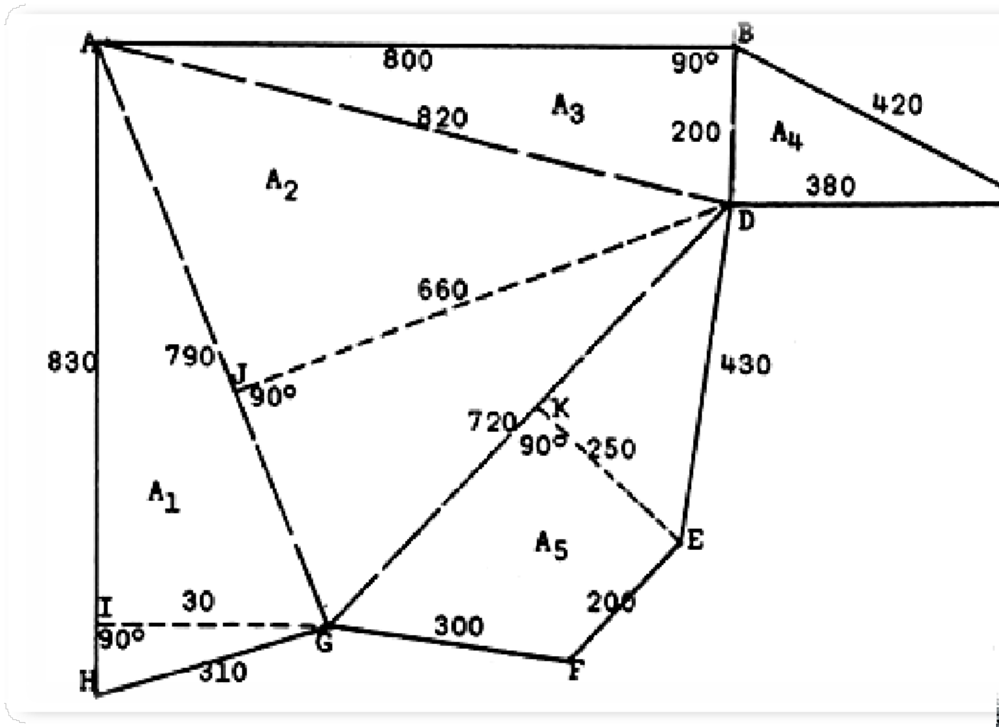
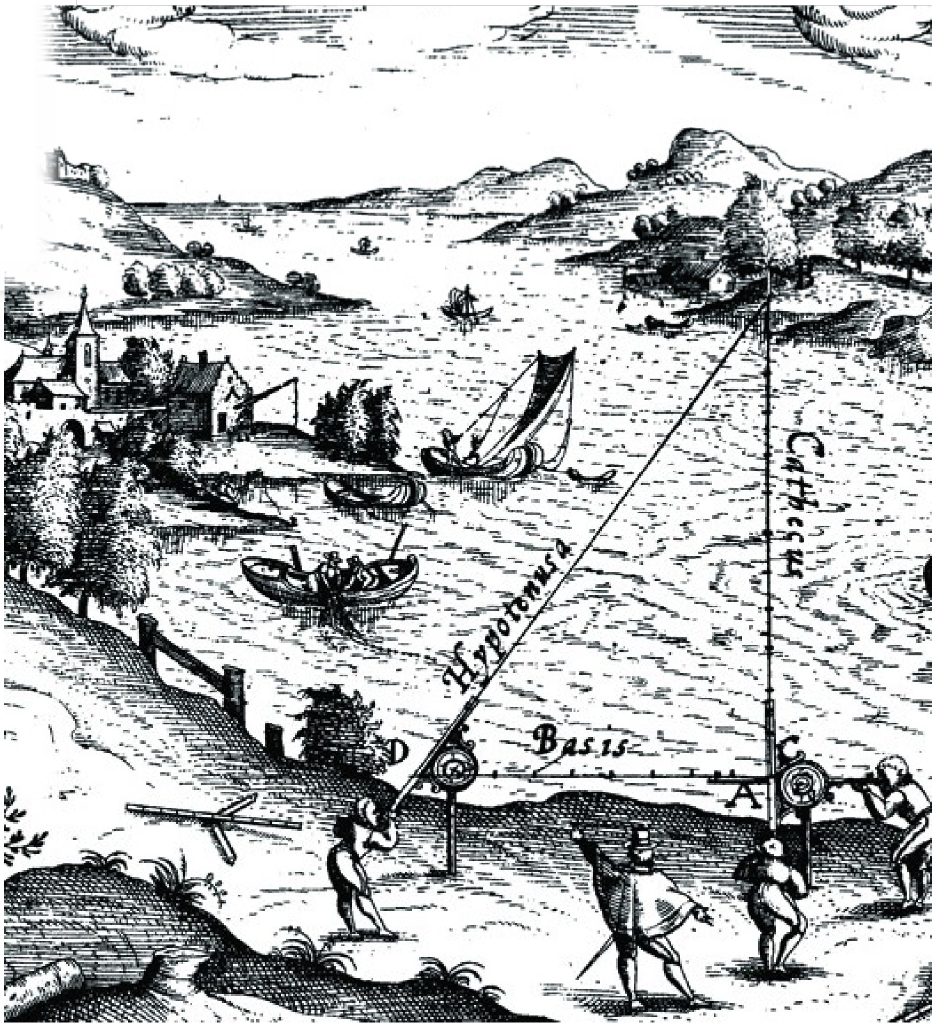

C
BCA
CAB
A
Baseline
ABC
B

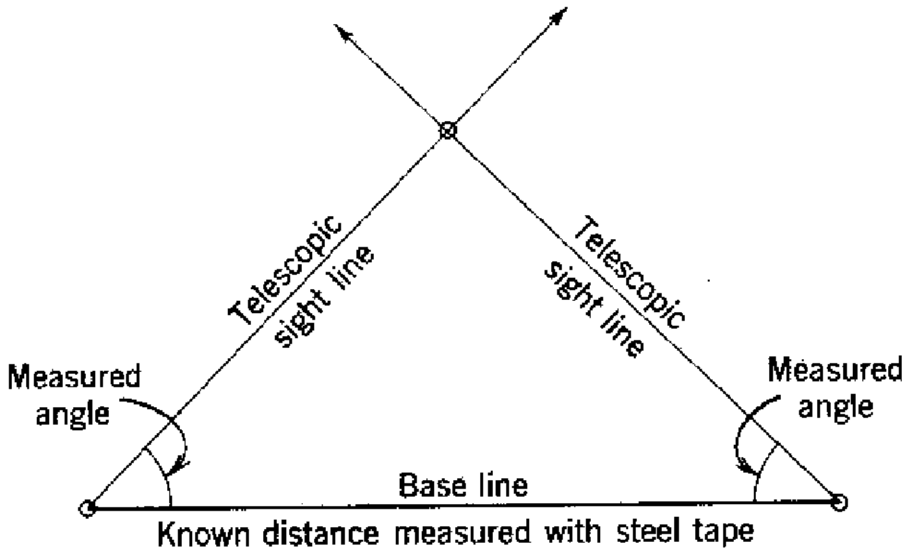
Figure 6-5. Distance angles of a single triangle

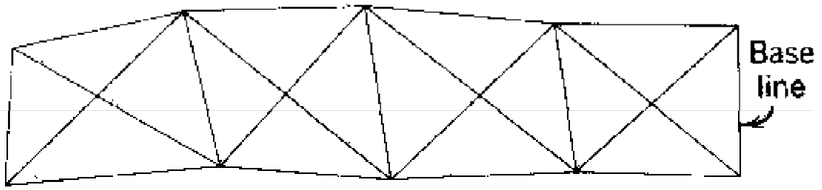

A
∡ 950 MILS
∡
1,500 MILS
DISTANCE ANGLES
∡ 750 MILS
BASE 2,000 METERS
C
AZIMUTH

Note. Double line indicates known base.


A
B
C
D
E
F
G
H
I
J
K
800
820
90°
200
420
380
A3
A4
A2
660
830
790
90°
430
720
90°
250
A1
A5
30
90°
200
300
310

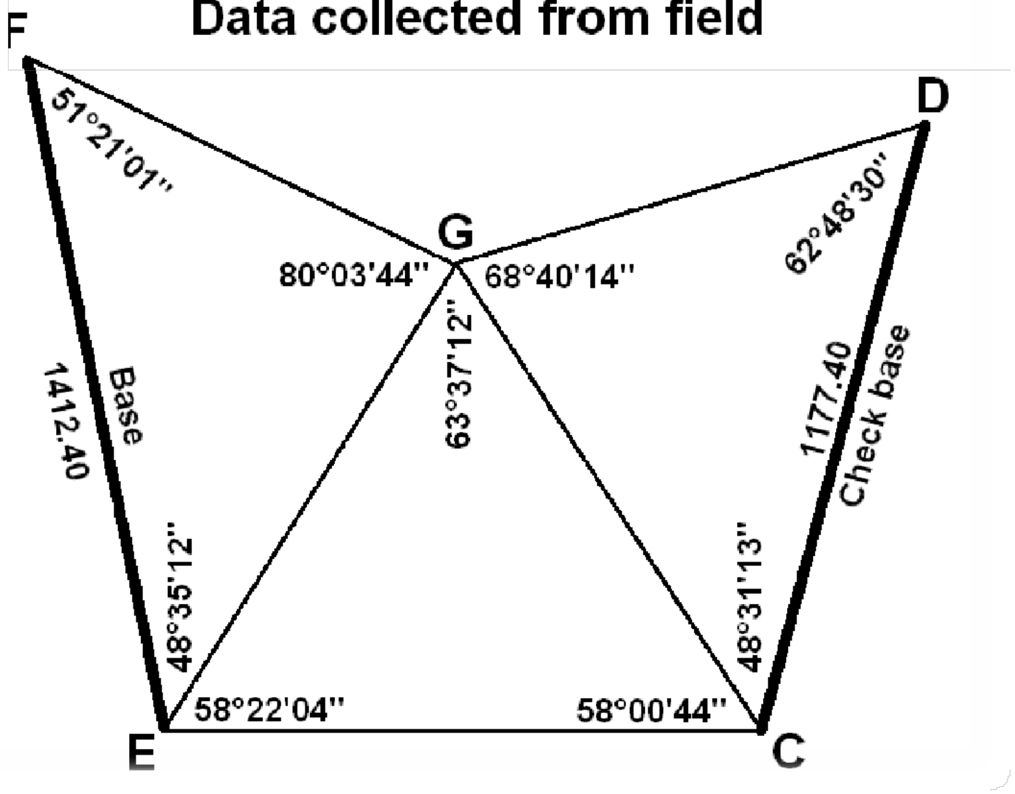

Cathetus
Hypotenusa
Basis
D
C
A

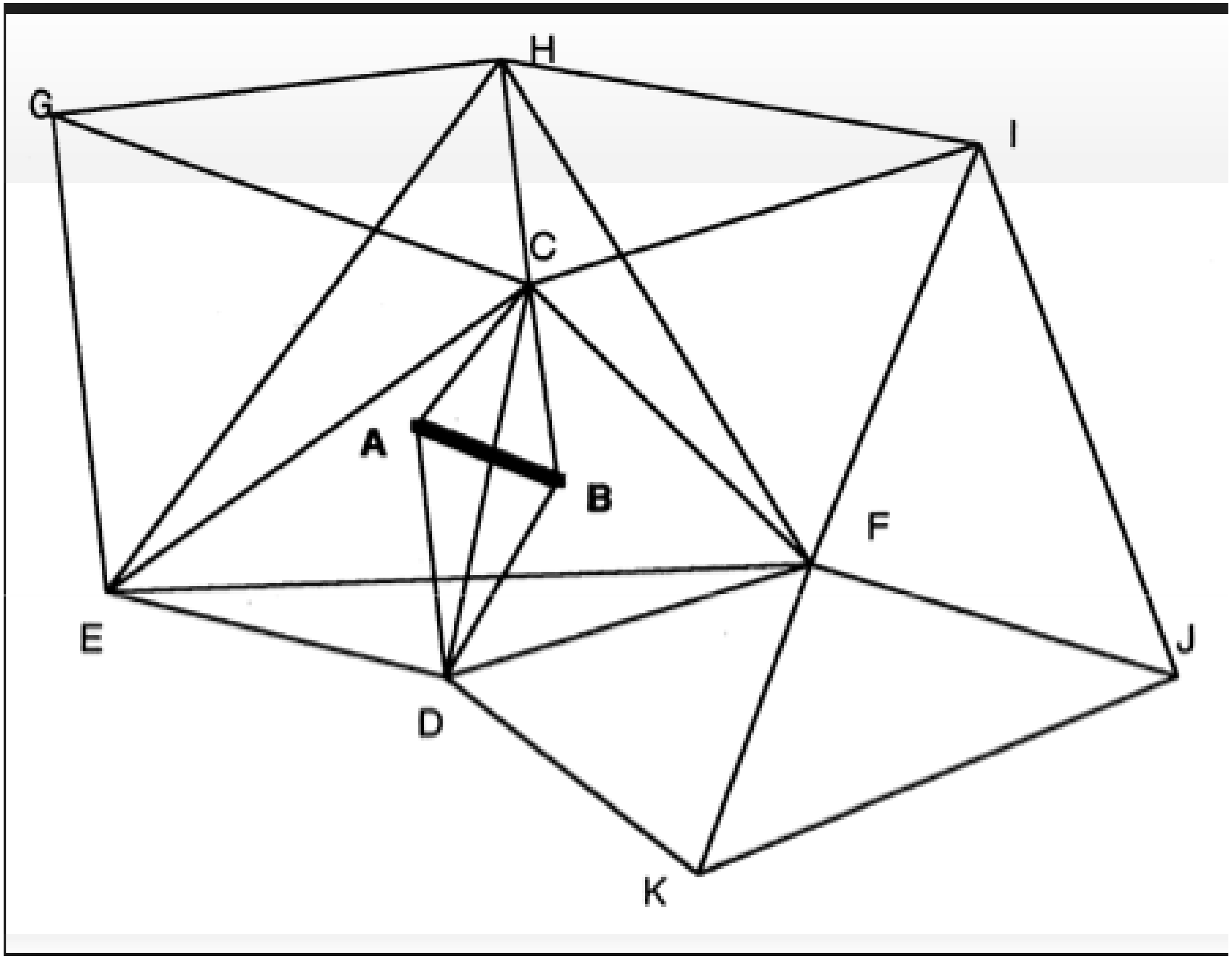

Telescopic sight line
Telescopic sight line
Measured angle
Measured angle
Base line
Known distance measured with steel tape



System of quadrilaterals

Data collected from field


F
D
G
E
C
51°21'01"
80°03'44"
68°40'14"
62°48'30"
63°37'12"
1412.40
Base
1177.40
Check base
48°35'12"
48°31'13"
58°22'04"
58°00'44"



- Triangulation using AB as a base line.
- Distance AB is measured precisely.

- Then C, D, E, F, G, H, I, J and K can be fixed by angular measurement only.

Triangulation and its Principles

- In triangulation all the three angles of each triangle are measured in the field along with one baseline.
- The side of the first triangle whose length is predetermined is called the base line and vertices of the individual triangles are known as triangulation stations and the whole figure is called the triangulation system or triangulation figure.
- The length and azimuth of each line is based on the length and azimuth of preceding line.
- To minimize accumulation of errors in lengths, subsidiary bases at suitable intervals are provided
- To control errors in azimuth of stations, astronomical observations are made at intermediate stations.
- The triangulation stations at which astronomical observations for azimuth are made are called Laplace station.

The formulae used to compute the angles from the measured sides are

$$a^2 = b^2 + c^2 - 2bcCosA$$

$$b^2 = c^2 + a^2 - 2caCosB$$

$$c^2 = a^2 + b^2 - 2abCosC$$

Or

$SinA = \frac{2\Delta}{bc}$, $SinB = \frac{2\Delta}{ca}$ and $SinC = \frac{2\Delta}{ab}$

Where

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)},$$

$$s = \frac{1}{2}(a+b+c)$$

Or

$$Tan\frac{A}{2} = \sqrt{s(s-a)\Big/(s-b)(s-c)}$$

$$Tan\frac{B}{2} = \sqrt{s(s-b)\Big/(s-c)(s-a)}$$

$$Tan\frac{C}{2} = \sqrt{s(s-c)\Big/(s-a)(s-b)}$$

Where

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)},$$

$$s = \frac{1}{2}(a+b+c)$$

Formula to compute co-ordinate of vetices

$$E_c = \frac{1}{2}(E_A + E_B) + \frac{a^2 - b^2}{2c^2}(E_A - E_B) - \frac{2\Delta}{c^2}(N_A - N_B)$$

$$N_c = \frac{1}{2}(N_A + N_B) + \frac{a^2 - b^2}{2c^2}(N_A - N_B) + \frac{2\Delta}{c^2}(E_A - E_B)$$

Where

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)}, \quad s = \frac{1}{2}(a+b+c)$$

Purpose of Triangulation Surveys

- Triangulation surveys are carried out for:

1. Establishment of accurate control points for plane and geodetic surveys of large areas, by ground methods.
2. Establishment of accurate control points for photogrammetric surveys of large areas.
3. Accurate location of engineering works i.e.

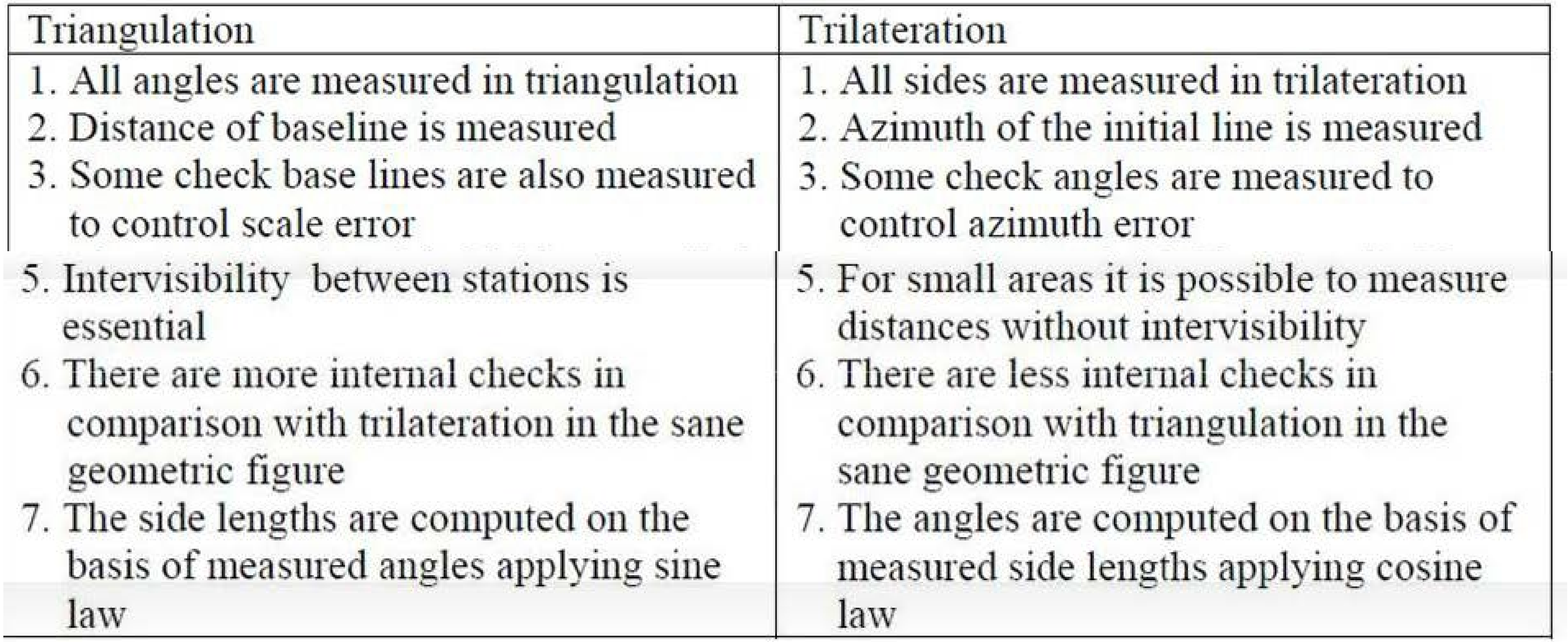
a. Fixing the centre line, terminal points and shafts for long tunnels,

b. Fixing centre line and abutments of long bridges over large rivers

c. Transferring the control points across wide sea channels, large water bodies etc.

d. Finding the direction of the movements of cloud.

Triangulation vs. Trilateration



Triangulation	Trilateration
1. All angles are measured in triangulation	1. All sides are measured in trilateration
2. Distance of baseline is measured	2. Azimuth of the initial line is measured
3. Some check base lines are also measured to control scale error	3. Some check angles are measured to control azimuth error
5. Intervisibility between stations is essential	5. For small areas it is possible to measure distances without intervisibility
6. There are more internal checks in comparison with trilateration in the sane geometric figure	6. There are less internal checks in comparison with triangulation in the sane geometric figure
7. The side lengths are computed on the basis of measured angles applying sine law	7. The angles are computed on the basis of measured side lengths applying cosine law

Classification of Triangulations

- The basis of the classification of triangulation figures is the accuracy with which the length and azimuth of a line of the triangulation are determined.
- On the basis of quality , accuracy & purpose, triangulations are classified as:

1.Primary or First order Triangulation
2. Secondary or Second order Triangulation
3.Tertiary or Third order Triangulation

Primary or First order Triangulation:

- Is the highest grade of triangulation system.
- To determine the shape & size of earth surface or to provide precise planimetric control points to which subsidiary triangulations may be connected.
- Stations of first order triangulation are generally selected 16 to 150 Km apart.
- Every possible precaution is taken in making linear, angular and astronomical observations, and also in their computation.

2. Secondary or Second order Triangulation:

- The secondary triangulation consists of a number of points fixed within the framework of primary triangulation.
- To provide control points closer together than those of primary.
- Secondary is classified, If primary doesnot attain standard of accuracy.
- The stations are fixed at close intervals so that the sizes of the triangles formed are smaller than the primary triangulation.

3. Tertiary or Third order Triangulation:

- Employed to provide control points between stations of primary & second order series.
- The third order triangulation consists of a number of points fixed within the framework of secondary triangulation, and forms the immediate control for detailed engineering and other surveys.
- The sizes of the triangles are small and instrument with moderate precision may be used.
- For topogaphical details, tertiary triangulations forms immediate control points.

STRENGTH OF FIGURE

- The strength of figure is a factor to be considered in establishing a triangulation system to maintain the computations within a desired degree of precision.
- It plays also an important role in deciding the layout of a triangulation system.
- U.S. Coast and Geodetic Surveys has developed a convenient method of evaluating the strength of a triangulation figure.
- It is based on the fact that computations in triangulation involve use of angles of triangle and length of one known side. The other two sides are computed by sine law.
- For a given change in the angles, the sine of small angles change more rapidly than those of large angles.
- This suggests that smaller angles less than 30° should not be used in the computation of triangulation.

Layout of Triangulation

- The arrangement of the triangles of a series is known as the layout of triangulation.

A series of triangulation may consists of:

1. Simple triangles in chain

- This layout of triangulation is generally used when control points are provided in narrow strip of terrain such as valley between ridges.
- Rapid and economical due to less number of stations to be sighted

  Do not provide independent check
- Check base lines and astronomical observations for a azimuth at frequent intervals

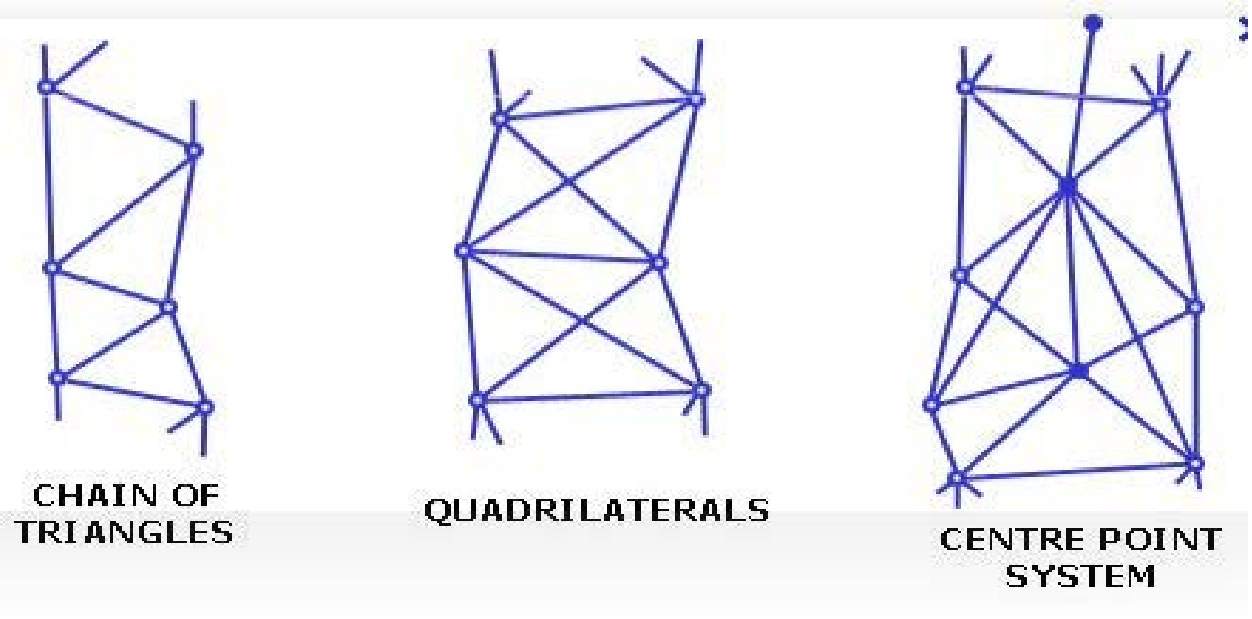
2. Braced quadrilaterals

- It consists of figures containing four corner stations and observed diagonals
- Best arrangement of triangles as it provides a means of computing the lengths of sides using different combination of sides and angles

..

3. Centered triangles and polygons

- It consists of figures containing centered polygons and centered triangles
- Used when vast area in all dimensions is required to be covered.
- Figures are quadrilaterals, pentagons or hexagons with central stations.



CHAIN OF TRIANGLES

QUADRILATERALS

CENTRE POINT SYSTEM

Satellite Stations

- To secure well-conditioned triangles or to have good visibility, objects such as chimneys, flat poles, towers, lighthouse, etc. are selected as triangulation stations.
- Such stations can be sighted from other stations but it is not possible to occupy the station directly below such excellent targets for making the observations by setting up the instrument over the station point.
- Also, signals are frequently blown out of position, and angles read on them have to be corrected to the true position of the triangulation station. Thus, there are two types of problems:

1. When the instrument is not set up over the true station
2. When the target is out of position.

- In Fig. 1.39, A, B, and C are the three triangulation stations.
- It is not possible to place instrument at C.
- To solve this problem another station S, in the vicinity of C, is selected where the instrument can be set up, and from where all the three stations are visible for making the angle observations.
- Such station is known as satellite station.
- As the observations from C are not possible, the observations form S are made on A, B, and, C from A and B on C.
- From the observations made, the required angle ACB is calculated. This is known as reduction to centre.

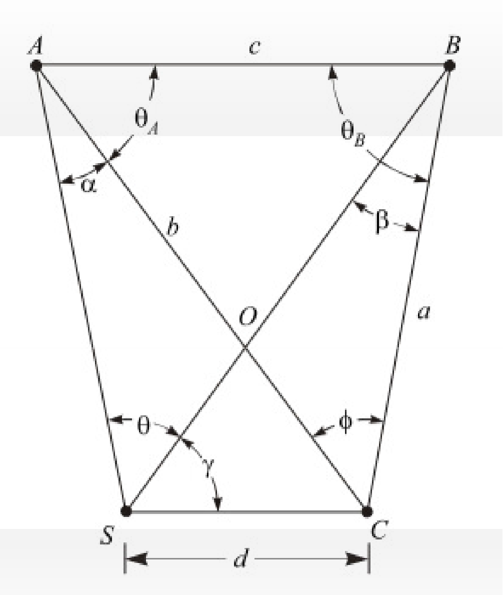


Fig. 1.39 Reduction to centre

Criteria for selection of triangulation stations

- Triangulation stations should be intervisible. For this purpose the station points should be on the highest ground such as hill tops, house tops, etc.
- Stations should be easily accessible with instruments.
- Station should form well-conditioned triangles.
- Stations should be so located that the lengths of sights are neither too small nor too long. Small sights cause errors of bisection and centering. Long sights too cause direction error as the signals become too indistinct for accurate bisection.
- Stations should be at commanding positions so as to serve as control for subsidiary triangulation, and for possible extension of the main triangulation scheme.
- Stations should be useful for providing intersected points and also for detail survey.
- Stations should be selected such that the cost of clearing and cutting, and building towers, is minimum.
- Grazing line of sights should be avoided, and no line of sight should pass over the industrial areas to avoid irregular atmospheric refraction.

Intervisibility of Traingulation Stations

- To execute reconnaissance for triangulation, either of the following general methods or a combination are used .
- In the first method, which can be used in hilly or mountainous country, test the intervisibility of the stations by visiting each one.
- If the stations are notintervisible from the ground, clear obstructions on the line (such as trees) or adjust the tower heights to allow intervisibility from either or both ends of the line. Find the elevations of intervening high pointsby lowering a weighted tape from a helicopter or by using barometric or trigonometric leveling.
- In thesecond method, obtain the elevations of the stations and the intervening country from maps or othersources, and determine the intervisibility of points and the required heights of towers from the data.
- Inactual practice, a combination of the two methodsis generally used.
- The reconnaissance party should keep a total tower height for any one line to a minimum. Determining the instrument and signal heights is a matter of good judgment, trial computations, and experience

TRAVERSING

Traversing

- A series of connected straight lines each joining two points on the ground, is called a ‘*traverse*’. End points are known as traverse stations & straight lines between two consecutive stations, are called traverse legs.

- A traverse survey is one in which the framework consists of a series of connected lines, the lengths and directions of which are measured with a chain or a tape, and with an angular instrument respectively.

- Traverses may be either a closed traverse or an open traverse:

1. **Closed Traverse:** A traverse is said to be closed when a complete circuit is made, i.e. when it returns to the starting point forming a closed polygon or when it begins and ends at points whose positions on plan are known. The work may be checked and “balanced”. It is particularly suitable for locating the boundaries of lakes, woods, etc. and for the survey of moderately large areas.
2. **Open Traverse:** A traverse is said to be open or unclosed when it does not form a closed polygon. It consists of a series of lines extending in the same general direction and not returning to the starting point. Similarly, it does not start and end at the points whose positions on plan are known. It is most suitable for the survey of a long narrow strip of country e.g. the valley of a river, the coast line, a long meandering road, or railway, etc.

Traversing

❑ **Important Technical Terms:**

1. **Meridians and Bearings:** The direction of survey lines may be defined in two ways:

a). Relatively to each other

b). Relatively to some reference direction

In the first case, directions are expressed in terms of the angles between two consecutive lines. In second case, these are expressed in terms of bearings.

The fixed direction on the surface of the earth with reference to which, bearings of survey lines are expressed is called a **meridian**. The meridians of reference directions employed in surveying may be one of the following:

a). True Meridian	b). Magnetic Meridian
c). Grid Meridian*	d). Arbitrary Meridian

o The line of intersection of the earth surface by a plane containing north pole, south pole and the given place is called **true meridian** or **geographical meridian**.

oThe geometrical longitudinal axis of a freely suspended & properly balanced magnetic needle, unaffected by local attractive forces, defines the magnetic north-south line which is called the **magnetic meridian**.

o The convenient direction assumed as meridian for measuring bearings of survey lines is known as arbitrary meridian.

Traversing

The horizontal angle between the reference meridian and the survey line measured in a clockwise direction is called **bearing**.

a). True Bearing b). Azimuth
c). Magnetic Bearing d). Grid Bearing

o The horizontal angle between the true meridian and a line measured in a clockwise direction is called **true bearing** of the line.
o The smaller angle which a survey line makes with true meridian is called **azimuth**.
oThe horizontal angle which a line makes with the magnetic meridian is called **magnetic bearing**.
o Bearings of survey lines referred to and reckoned from grid lines are called grid bearings.

o The angle between the true meridian & the magnetic meridian at any place is known as **magnetic declination**, whereas, the angle between the true meridian & the grid meridian at any place is known as **grid convergence**.

Traversing

- ☐ **Designation of Bearings:**
  1. The Whole Circle Bearing System (W.C.B)
  2. The Quadrantal Bearing System (Q.B)

1. **The W. C. B System:** The W. C. B system is also sometimes known as **Azimuthal System**. In this system, bearing of a line is measured from the true north or magnetic north in clockwise direction. The value of a bearing may vary from 0° to 360°, utilizing the whole circle of graduations. Prismatic Compass is graduated on whole circle bearing system.

2. **The Q. B System:** In Q. B System, bearings of survey lines are measured eastward or westward from North and South whichever is nearer. In this system, both north and south directions are used as reference meridians & bearings are reckoned either clockwise or anticlockwise, depending upon the position of the line. The quadrant in which a line lies is mentioned to specify the location of the line. Surveyor's compass is graduated in quadrantal bearing system.

Bearings designated by Q.B. System are sometime called **Reduced Bearings**.

Traversing

Conversion Of W.C.B into Q.B

CASE	W.C.B between	Rule for Q.B	Quadrant
I	0˚ and 90˚	W.C.B	N.E.
II	90˚ and 180˚	180˚ - W.C.B	S.E.
III	180˚ and 270˚	W.C.B - 180˚	S.W.
IV	270˚ and 360˚	360˚ - W.C.B	N.W.

Traversing

Conversion Of Q.B into W.C.B

CASE	R.B	Rule for W.C.B	W.C.B between
I	N α° E	R.B	0˚ and 90˚
II	S β° E	180˚ - R.B	90˚ and 180˚
III	S γ° W	180˚+R.B	180˚ and 270˚
IV	N δ° W	360˚ - R.B	270˚ and 360˚

SURVEYING

- **Conversion of bearing from one system to the other:**

1. Convert the following whole circle bearing to quadrantal bearing:

a. 68^0 32’
b. 132^0 12’
c. 236^0 37’
d. 334^0 52’

2. Convert the following reduced bearing to whole circle bearings:

a. N 36^0 16’E
b. S 28^0 14’E
c. S 47^0 26’ W
d. N 58^0 24’ W

Traversing

❑ **FORE and BACK Bearings:**

Every line may be defined by two bearings, one observed at either end of the line. Both the bearings expressed in W.C.B System differ each other by 180°. The bearing of a line in the direction of the progress of survey, is called **Fore or Forward Bearing** (F.B) while the bearing in the opposite direction of the progress of survey is known as **Reverse or Back Bearing** (B.B).

❑**Relationship Between Fore and Back Bearings:**

a). W.C.B System:

Back bearing = Fore Bearing ± 180°

Positive sign is used when fore bearing is less than 180° and negative sign is used when the fore bearing is greater than 180°.

b). Q.B System:

To convert the fore bearing of a line into its back bearing in Q.B system, replace N by S, S by N, E by W and W by E , without changing the numerical value of the bearing.

Traversing

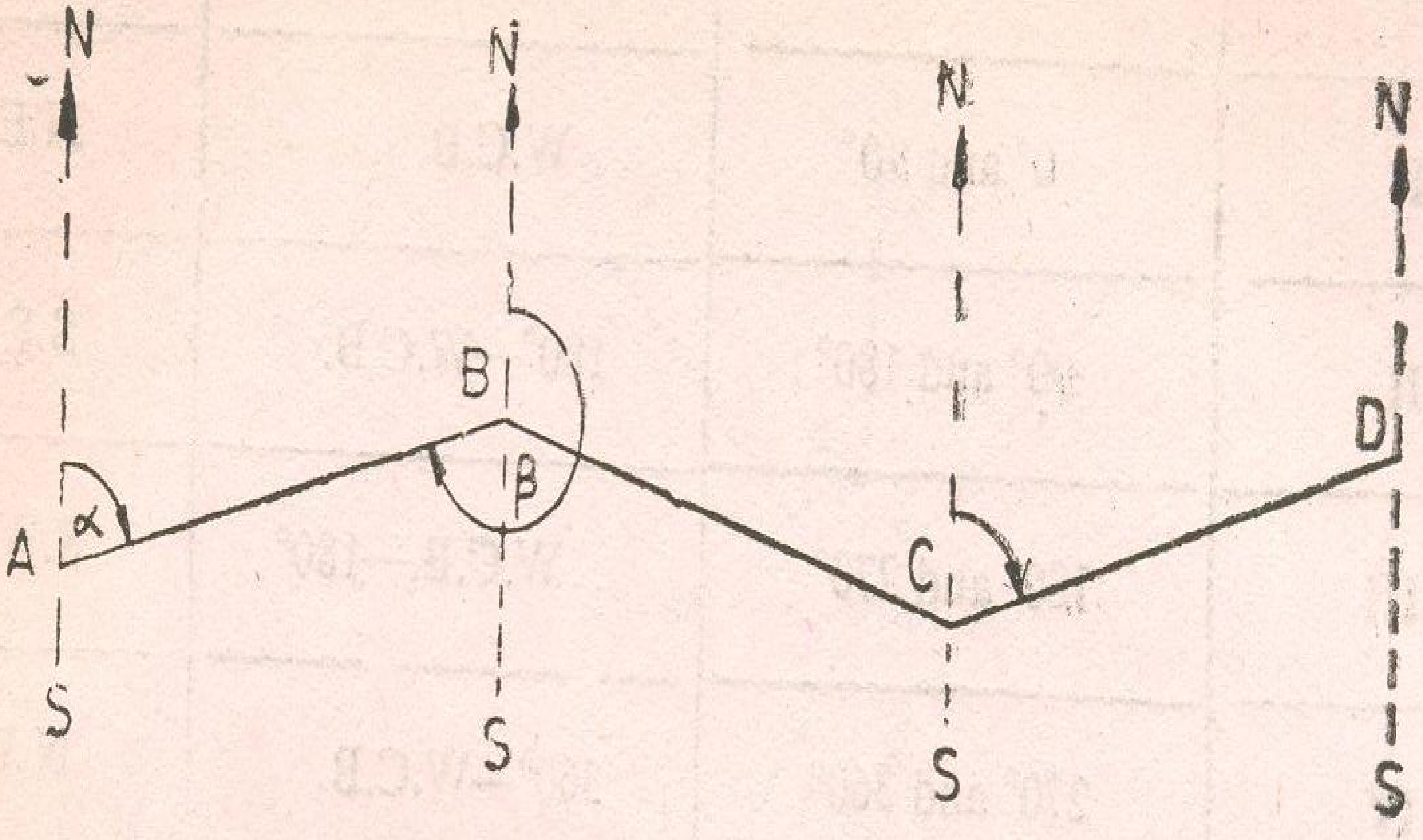

N
N
N
N
B
β
α
A
C
D
S
S
S
S

Fig. 4·21. Direction of survey is West to East

Traversing

1. The following are the observed fore bearing of the lines:
   a. AB, 38^0 14’ ; BC, 142^0 18’ ; CD, 208^0 37’ and DE, 318^0 26’
   b. Find their back bearings.

2. The fore bearings of the lines are as follows:
   a. AB: N 32^0 12’ E; BC: S 43^0 18 E; CD: S 26^0 30’ W; DE: N 65^0 24’ W.
   b. Find their back bearings

Traversing

- **Calculation of included angles from bearings:**

knowing the bearings of two adjacent lines, their included angles may be easily calculated as under:

a) Given W.C.B of lines:

let W.C.B of the line AB = α°

W.C.B of the line AC = β°

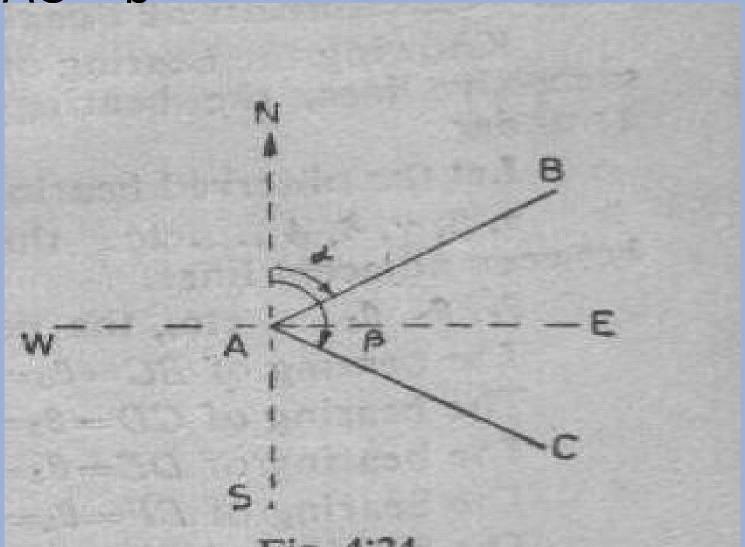


Fig. 4·24

Therefore , the included angle BAC = <NAC - <NAB

= β – α

= Bearing of AC – bearing of AB

Traversing

b) Given Q.B of lines:

A diagram may be drawn and bearings of the lines plotted in their respective quadrants. The included angle is calculated from one of the under mentioned formulae:

1) If the bearings have been measured to the same side of the common meridian, the included angle $\alpha = \theta_2 - \theta_1$ i.e. the difference of the bearings. This is true for all quadrants.[fig 4.25 a]
2) If the bearings have been measured to the opposite side of the common meridian, the included angle $\alpha = \theta_1 + \theta_2$ i.e. the sum of the bearings. [fig 4.25 b]
3) If the bearings have been measured to the same side of the different meridians, the included angle $\alpha = 180^\circ - (\theta_1 + \theta_2)$ i.e. the difference of 180 and the sum of the bearings. [fig 4.25 c]
4) If the bearings have been measured to the opposite side of the different meridians, the included angle $\alpha = 180^\circ - (\theta_1 - \theta_2)$ i.e. the difference of 180 and the difference of the bearings. [fig 4.25 d]

SURVEYING – I (CE- 128)

II. Given Q.B. of lines

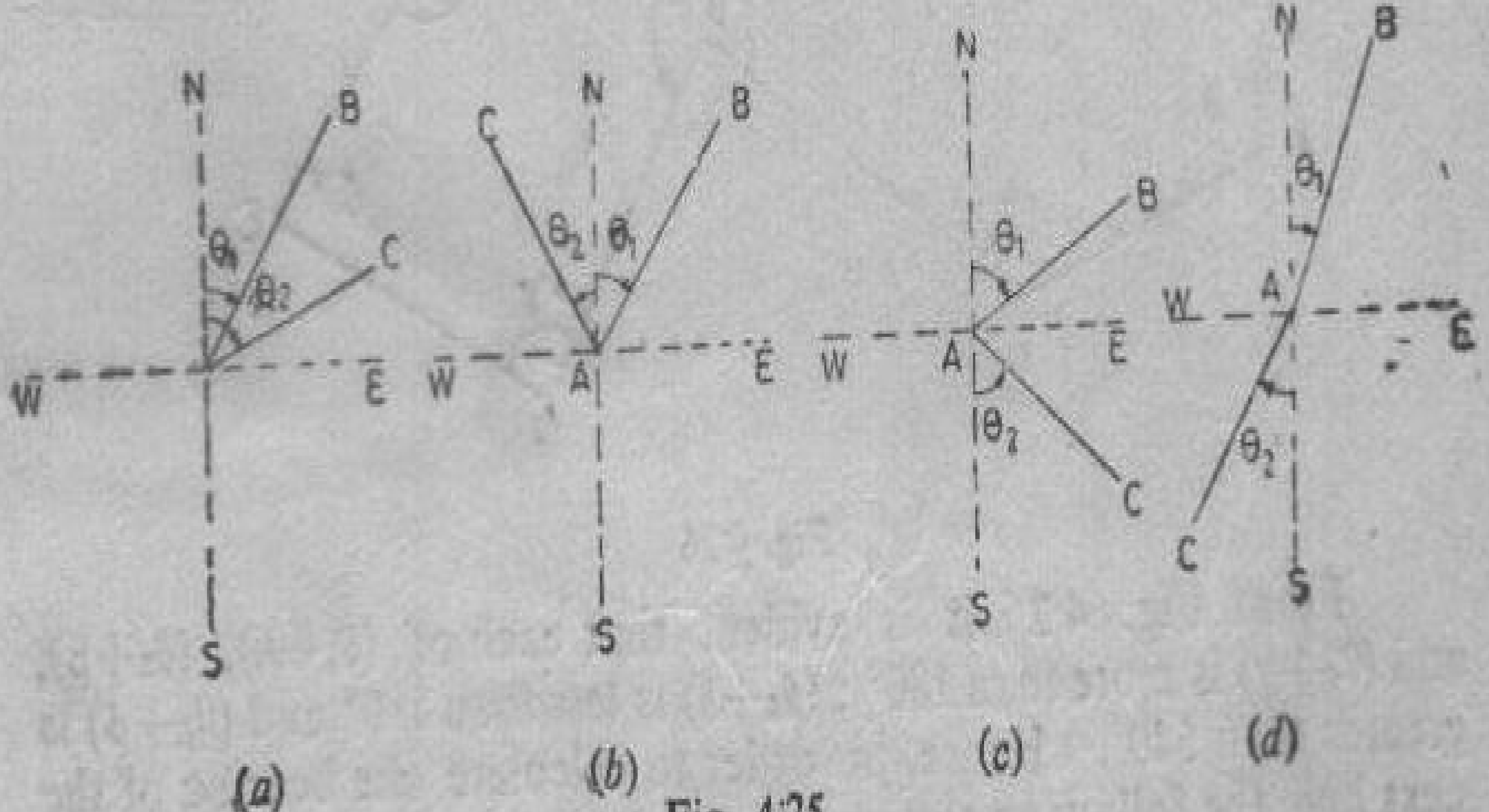

N
B
C
θ1
θ2
W
E
S
(a)
C
N
B
θ2
θ1
W
A
E
S
(b)
N
B
θ1
W
A
E
θ2
C
S
(c)
N
B
θ1
W
A
E
θ2
C
S
(d)

Fig. 4·25

Traversing

- **Calculation of bearings from included angles :**

Knowing the bearing of the line and the included angles between the successive lines, the bearing of the remaining lines, may be calculated as under:

Let the observed bearing of the line AB be θ_1 (given)

α , β , γ , δ , φ ………etc, the included angles measured clockwise between adjacent lines. θ_2 ,θ_3 ,θ_4 ,θ_5, etc , the bearing of successive lines.

The bearing of BC = $\theta_2 = \theta_1 + \alpha - 180^\circ$

The bearing of CD = $\theta_3 = \theta_2 + \beta - 180^\circ$

The bearing of DE = $\theta_4 = \theta_3 + \gamma - 180^\circ$

The bearing of EF = $\theta_5 = \theta_4 + \delta + 180^\circ$

The bearing of FG = $\theta_6 = \theta_5 + \varphi - 540^\circ$

From fig.4.26, it is evident that each of $(\theta_1 + \alpha)$, $(\theta_2 + \beta)$ and $(\theta_3 + \gamma)$ is more than 180°;

$(\theta_4 + \delta)$ is less than 180° and $(\theta_6 + \varphi)$ is greater than 540°.

Traversing

Hence, in order to calculate the bearing of the next line, the following statements may be made:

“***Add the included angle measured clockwise to the bearing of the previous line. If the sum is:***

more than 180˚,deduct 180˚

more than 540˚,deduct 540˚

less than 180˚,add 180˚,to get the bearing of the next line.”

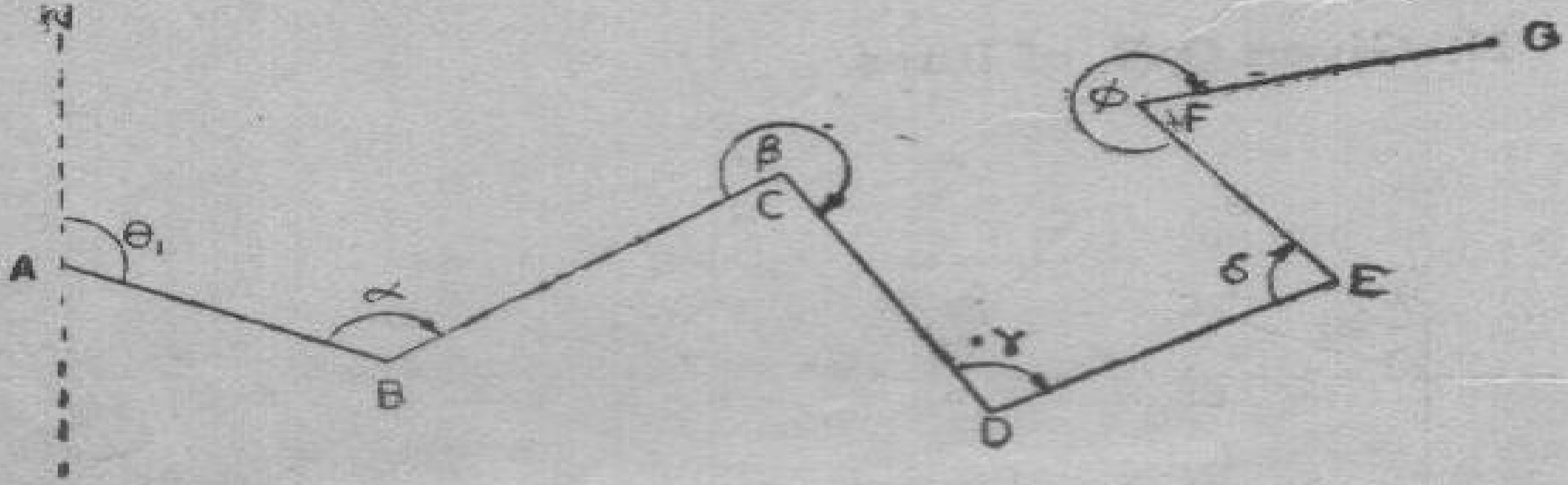


Fig. 4·26

Traversing

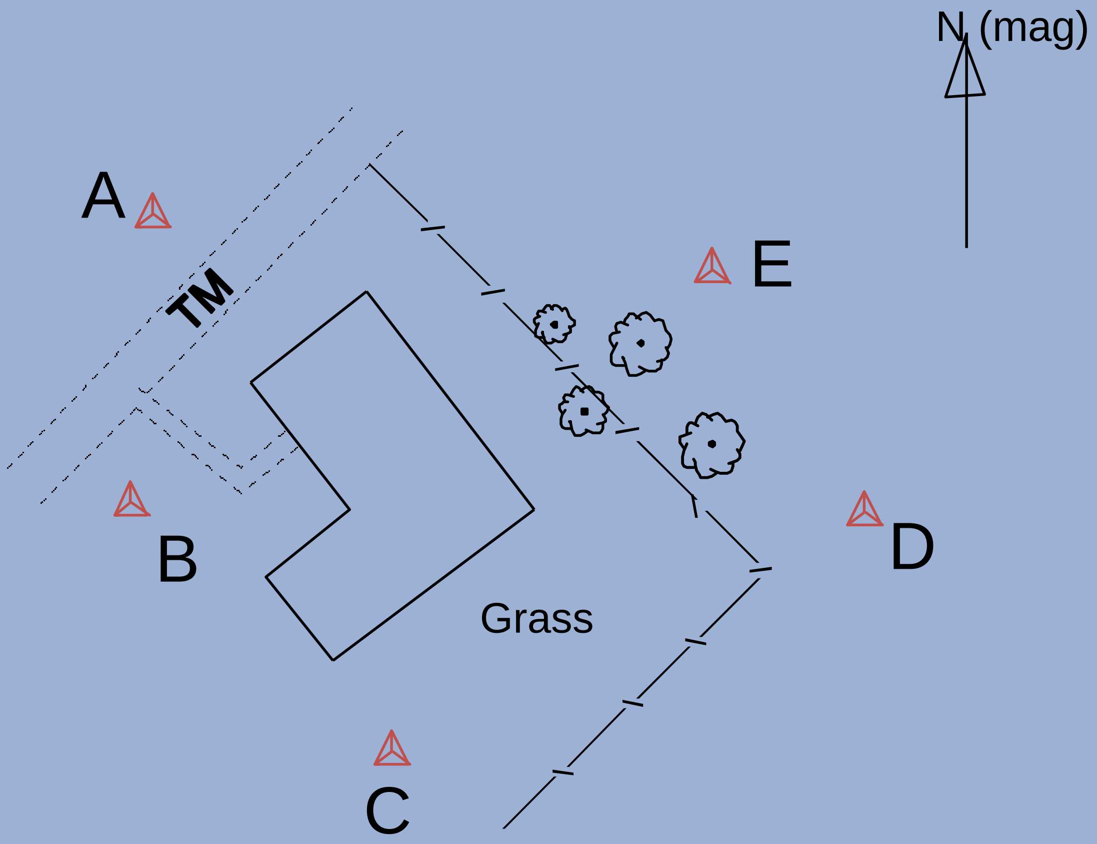
- ❑ **TRAVERSE EXAMPLE:**

Horizontal Control

- o Horizontal control is required for initial survey work (detail surveys) and for setting out.

- o The simplest form is a TRAVERSE - used to find out the co-ordinates of CONTROL or TRAVERSE STATIONS.

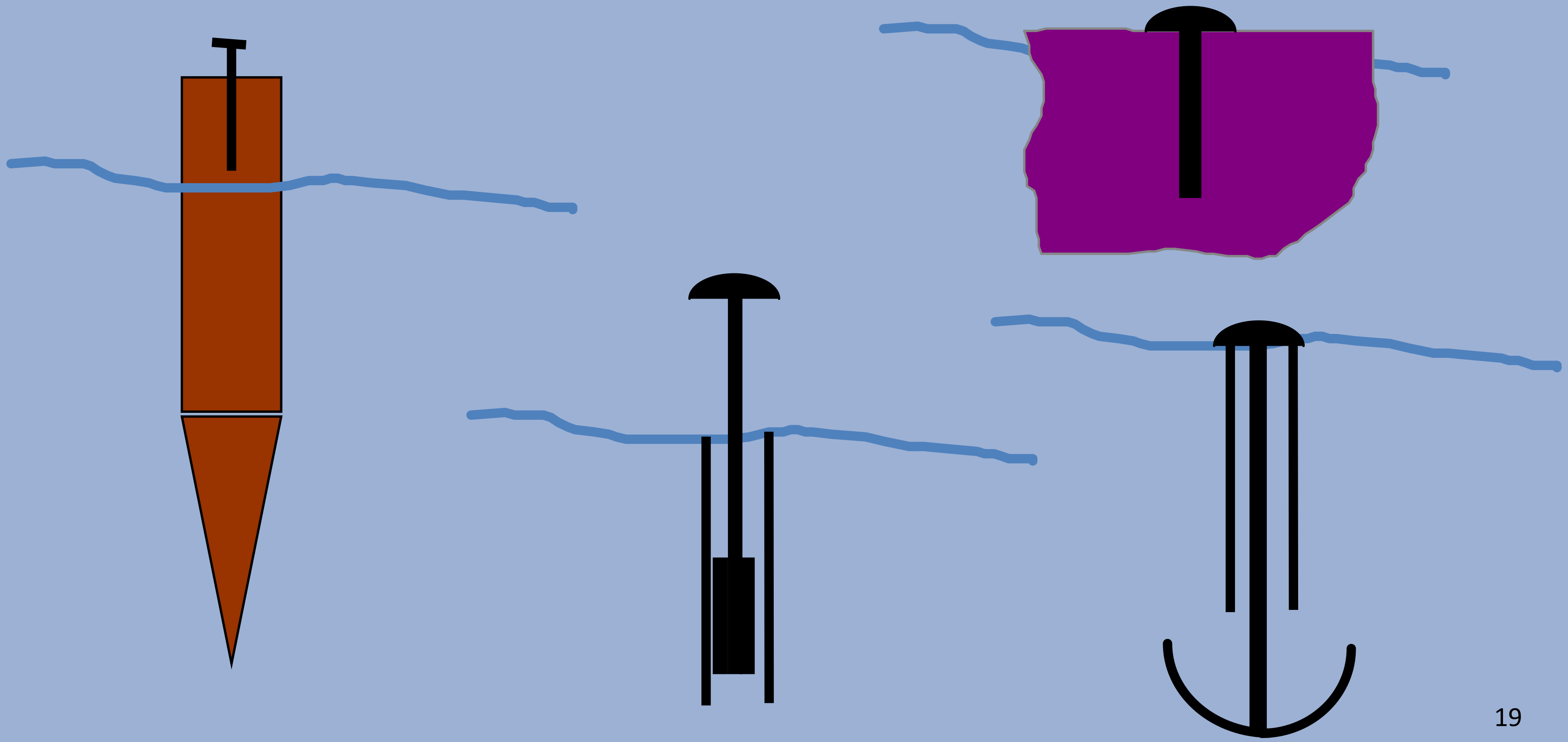
Traversing


N (mag)
A
E
TM
B
D
Grass
C

Traversing

Horizontal Control

- Horizontal control is required for initial survey work (detail surveys) and for setting out.
- The simplest form is a TRAVERSE - used to find out the co-ordinates of CONTROL or TRAVERSE STATIONS.

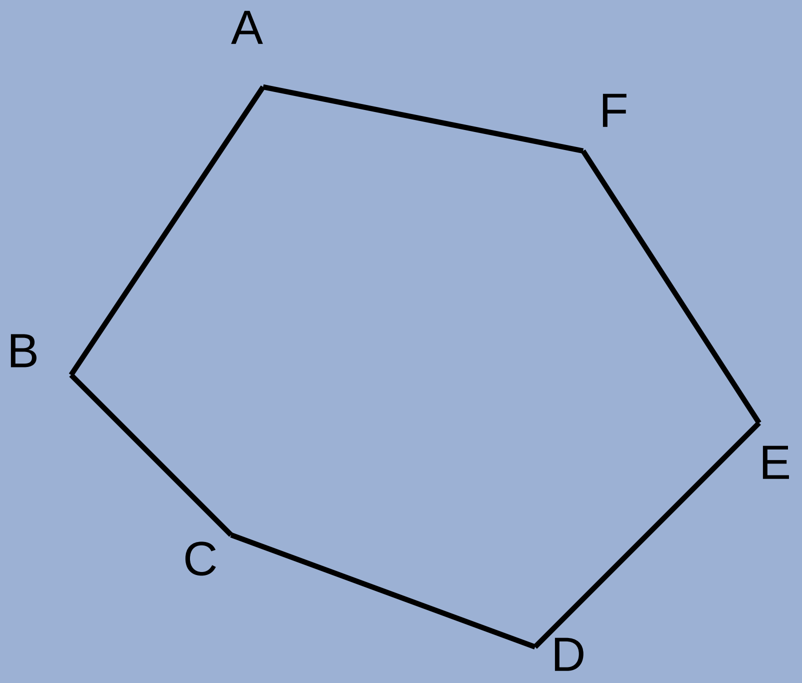


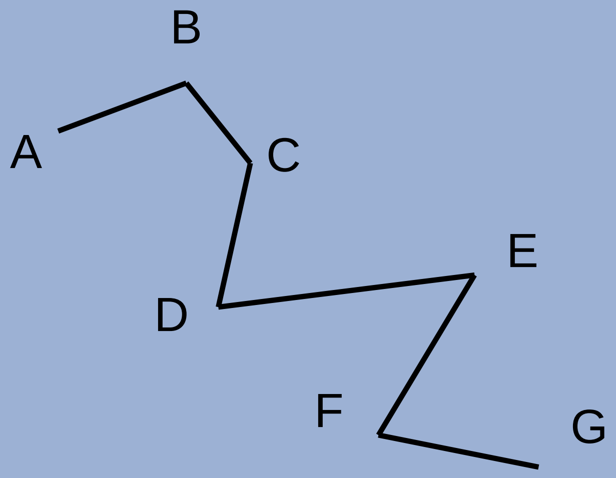
Traversing

- There are two types : -

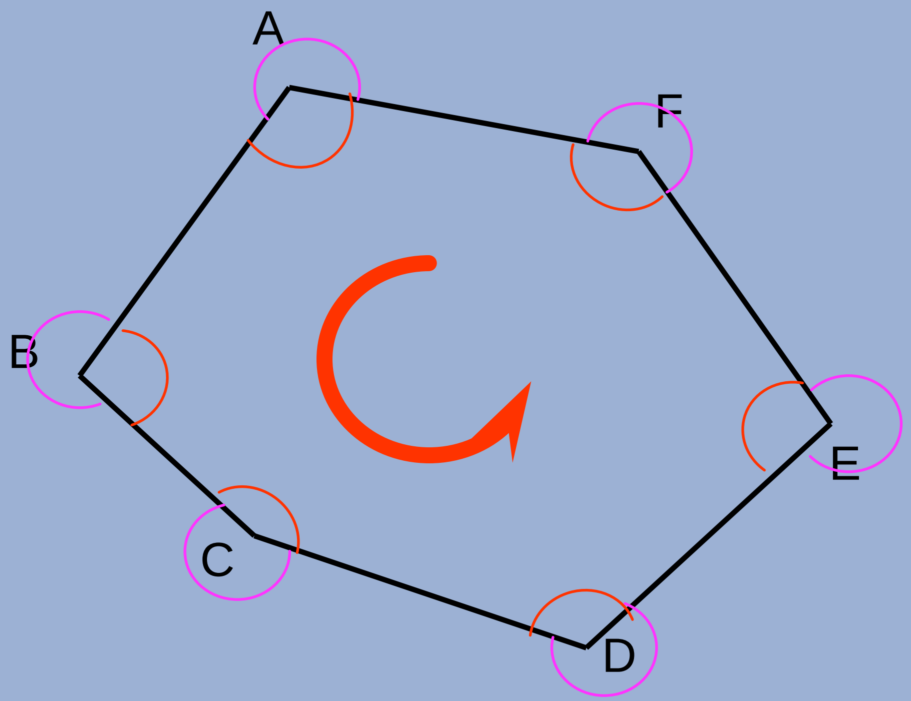
a) POLYGON or LOOP TRAVERSE

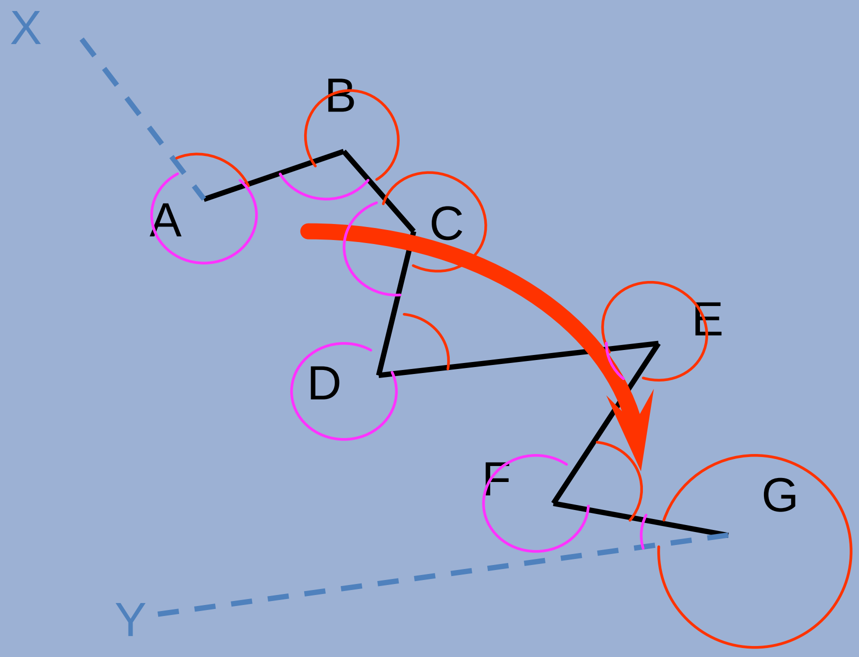
b) LINK TRAVERSE





Traversing





a) is obviously closed

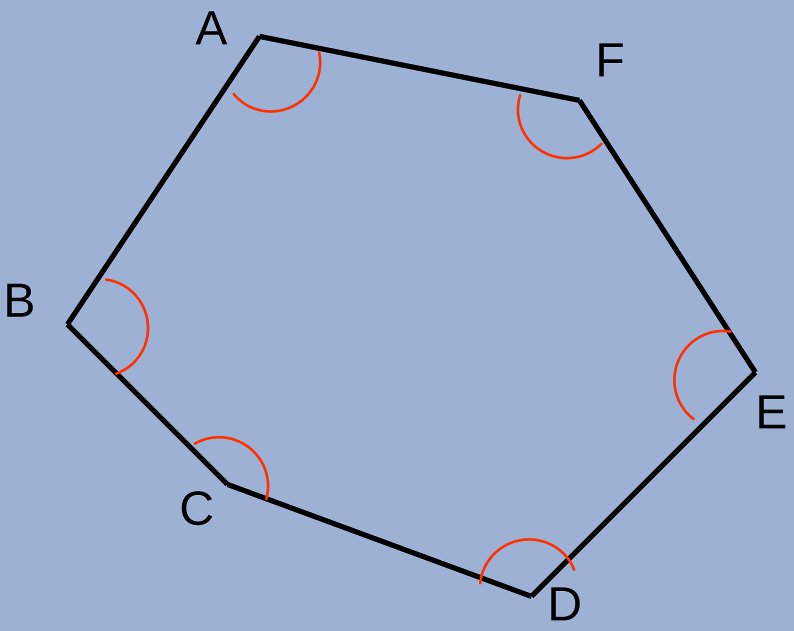
b) must start and finish at points whose co-ordinates are known,

and must also start and finish with angle observations to other known points.

- Working in the direction A to B to C etc is the **FORWARD DIRECTION**
- This gives two possible angles at each station.

LEFT HAND ANGLES **RIGHT HAND ANGLES**

Traversing



Consider the **POLYGON** traverse

The **L.H.Angles** are also the
INTERNAL ANGLES

Using a theodolite we can measure all the internal angles.

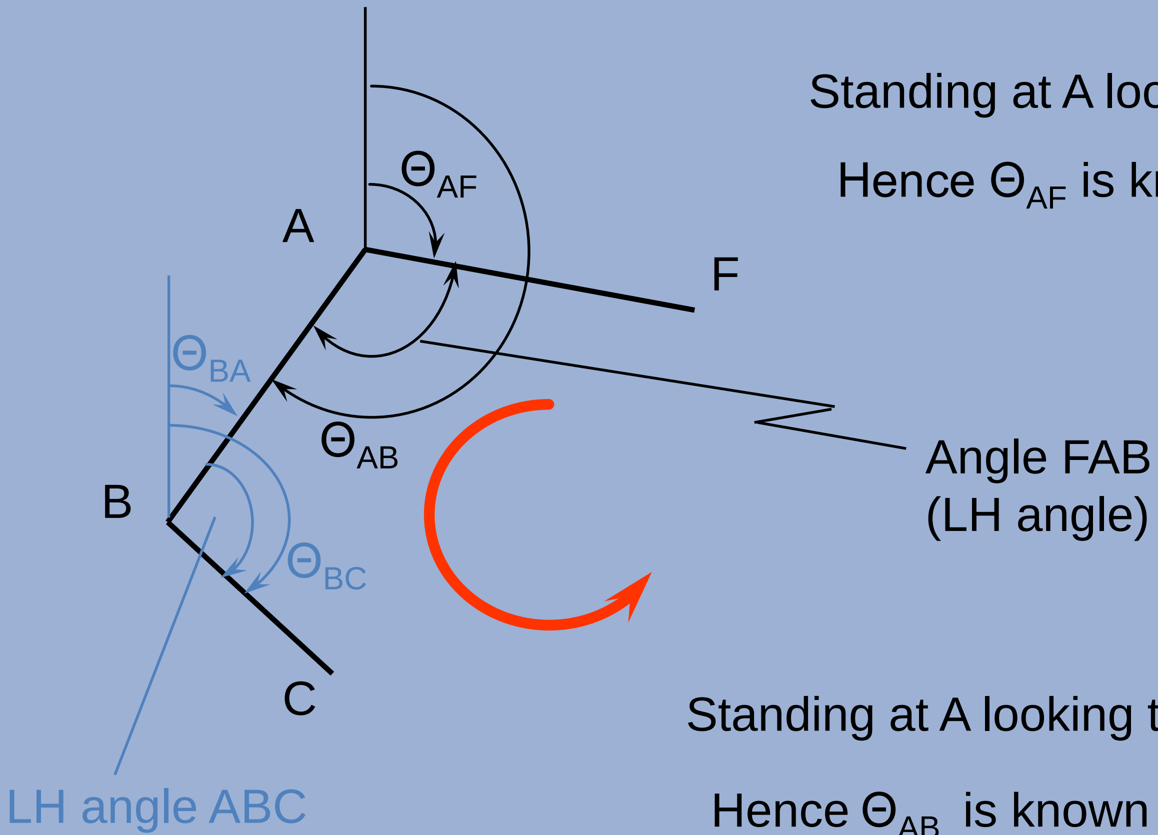
Σ (Internal Angles) = $(2N - 4) * 90^0$

The difference between
Σ Measured Angles and Σ Internal Angles
is the Angular Misclosure

Maximum Angular Misclosure = 2 (or 3) * Accuracy of Theodolite * √ (No. of Angles)

(Rule of thumb)

Traversing


A
F
B
C
Θ_{AF}
Θ_{AB}
Θ_{BA}
Θ_{BC}
Angle FAB
(LH angle)
LH angle ABC

Standing at A looking towards F - looking **BACK**

Hence Θ_{AF} is known as a **BACK BEARING**

Standing at A looking towards B - looking **FORWARD**

Hence Θ_{AB} is known as a **FORWARD BEARING**

BACK BEARING (Θ_{AF}) + L.H.ANGLE (<FAB)

= NEXT FORWARD BEARING (Θ_{AB})

Reminder: every line has two bearings

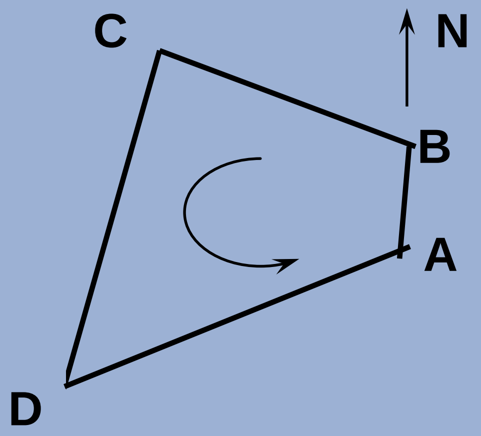
BACK BEARING (Θ_{BA}) = FORWARD BEARING (Θ_{AB}) 180^0

Traversing

Traverse Example

12” / 4 = 3”

Observations, using a Zeiss O15B, 6” Theodolite, were taken in the field for an anti - clockwise polygon traverse, A, B, C, D.



Traverse Station	Observed Clockwise Horizontal Angle 0 ‘ “
A	132 15 30 - 3”
B	126 12 54 - 3”
C	69 41 18 - 3”
D	31 50 30 - 3”

Line	Horizontal Distance
AB	638.57
BC	1576.20
CD	3824.10
DA	3133.72

Σ (Internal Angles) = 360 00 12

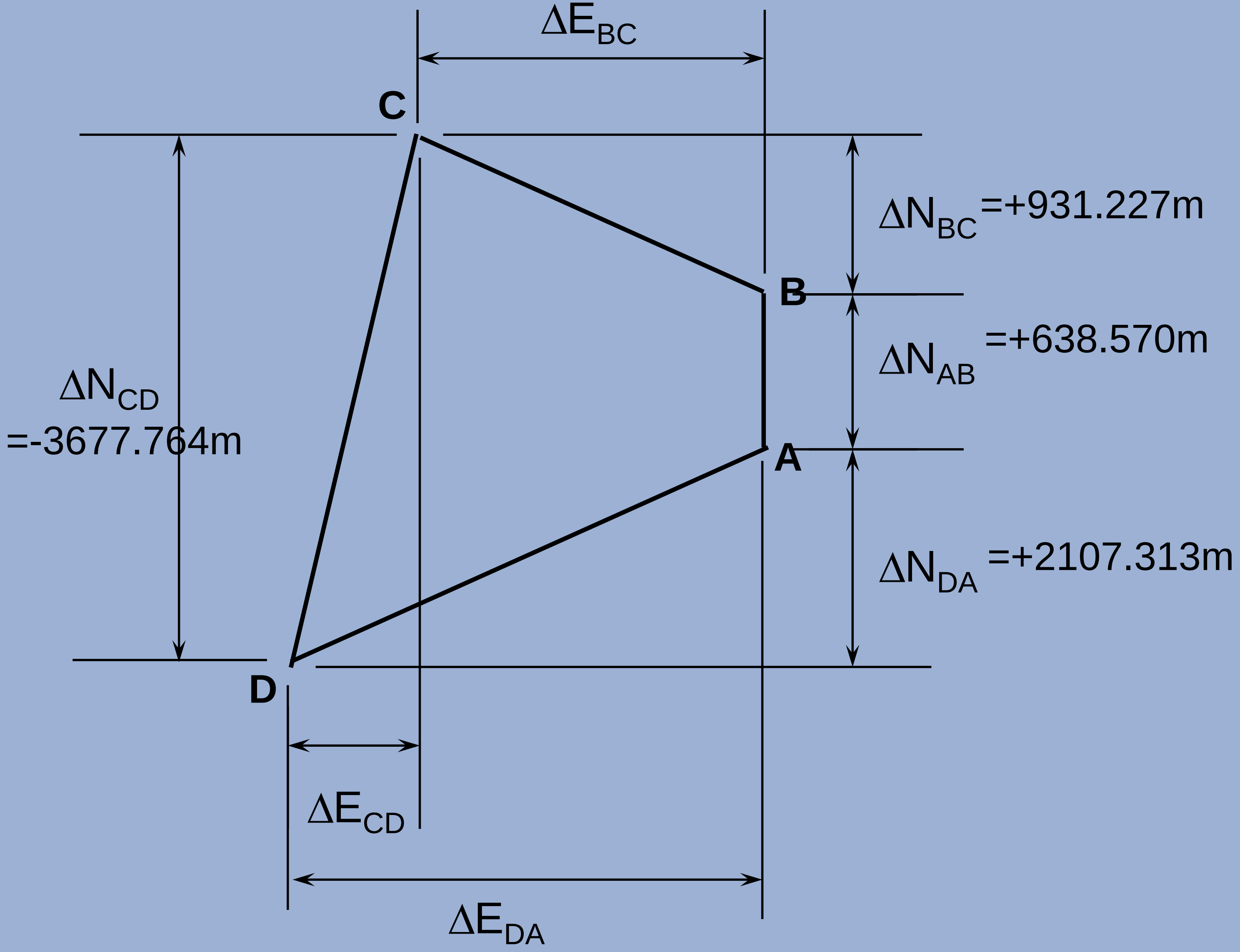
Σ (Internal Angles) should be (2N-4)*90 = 360 00 00

Allowable = 3 * 6” * √N= 36”

OK - Therefore distribute error

The bearing of line AB is to be assumed to be 0^0 and the co-ordinates of station A are (3000.00 mE ; 4000.00 mN)

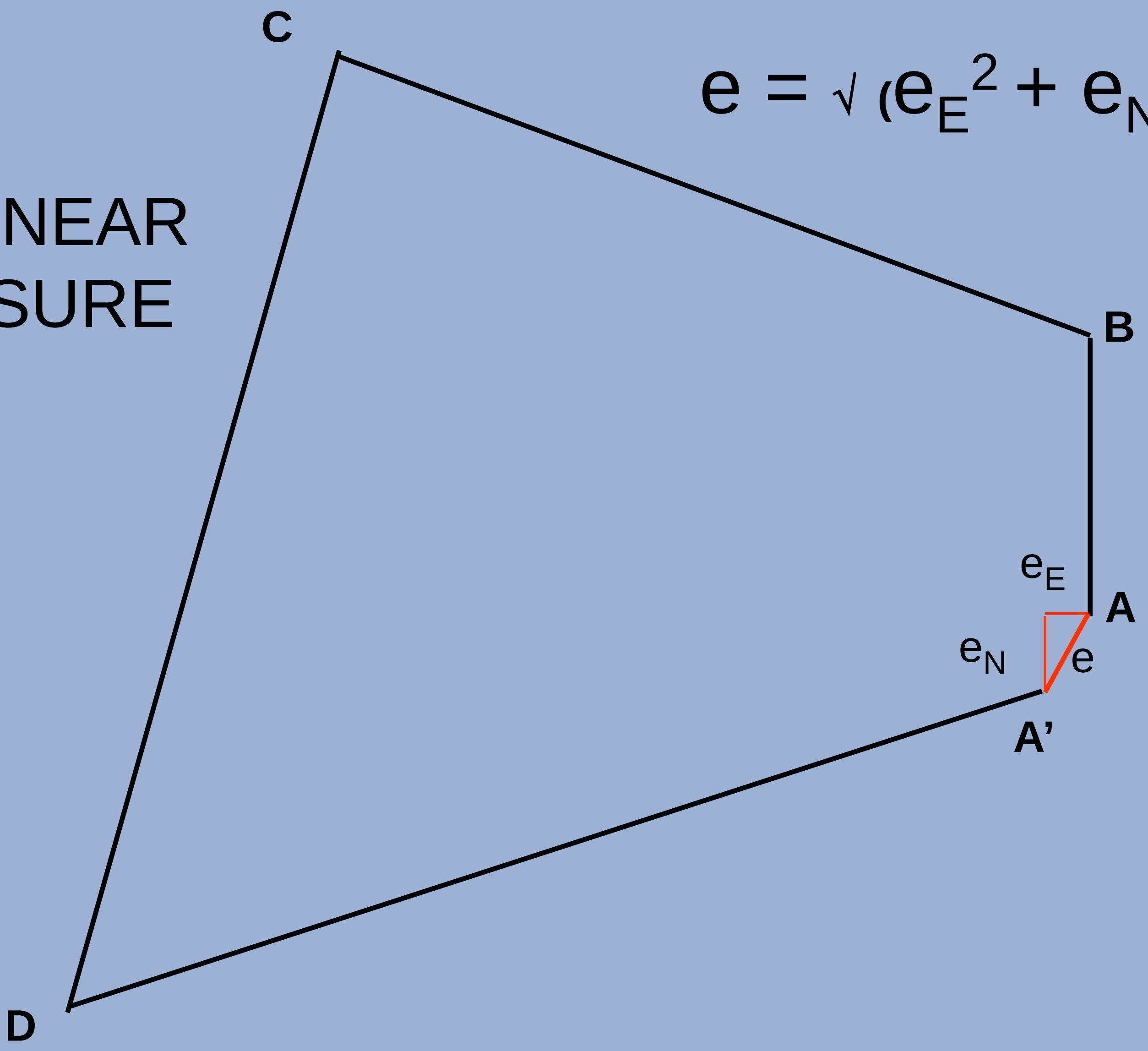
SURVEYING – I (CE- 128)


ΔE_{BC}
C
ΔN_{BC}=+931.227m
B
ΔN_{AB} =+638.570m
ΔN_{CD}
=-3677.764m
A
ΔN_{DA} =+2107.313m
D
ΔE_{CD}
ΔE_{DA}

Traversing

$$e = \sqrt{(e_E^2 + e_N^2)}$$

e is the LINEAR MISCLOSURE


C
B
e_E
A
e_N
e
A’
D

THEODOLITE SURVEYING

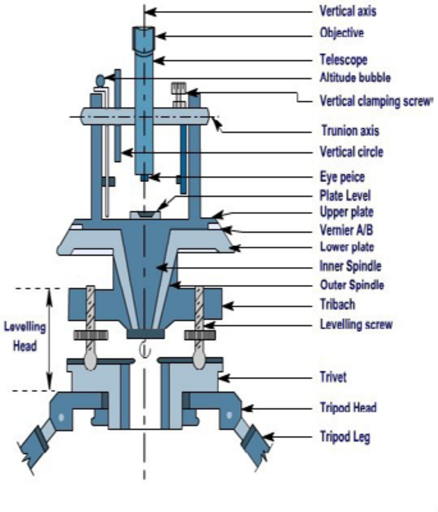

Vertical axis
Objective
Telescope
Altitude bubble
Vertical clamping screw
Trunion axis
Vertical circle
Eye peice
Plate Level
Upper plate
Vernier A/B
Lower plate
Inner Spindle
Outer Spindle
Tribach
Levelling screw
Trivet
Tripod Head
Tripod Leg
Levelling Head

Figure 20.4 Sectional view of a Thedolite

S. Maithya

INTRODUCTION:

So far we have been measuring horizontal angles by using a *Compass* with respect to *meridian*, which is *less accurate* and also it is not possible to measure vertical angles with a Compass.

So when the objects are at a considerable distance or situated at a considerable elevation or depression ,it becomes necessary to measure horizontal and vertical angles more precisely. So these measurements are taken by an instrument known as a *theodolite*.

THEODOLITE SURVEYING

The system of surveying in which the angles are measured with the help of a theodolite, is called Theodolite surveying.

CLASSIFICATION OF THEODOLITES

Theodolites may be classified as ;

A.BASE ON HORIZONTAL AXIS

i)Transit Theodolite.

ii)Non Transit Theodolite.

B.BASE ON ANGEL

i)Vernier Theodolites.

ii)Micrometer Theodolites.

iii)Electronic digital theodolite.

CLASSIFICATION OF THEODOLITES

A. **Transit Theodolite:** A theodolite is called a transit theodolite when its telescope can be transited i.e revolved through a complete revolution about its horizontal axis in the vertical plane, whereas in a-

Non-Transit type, the telescope cannot be transited. They are inferior in utility and have now become *obsolete.*

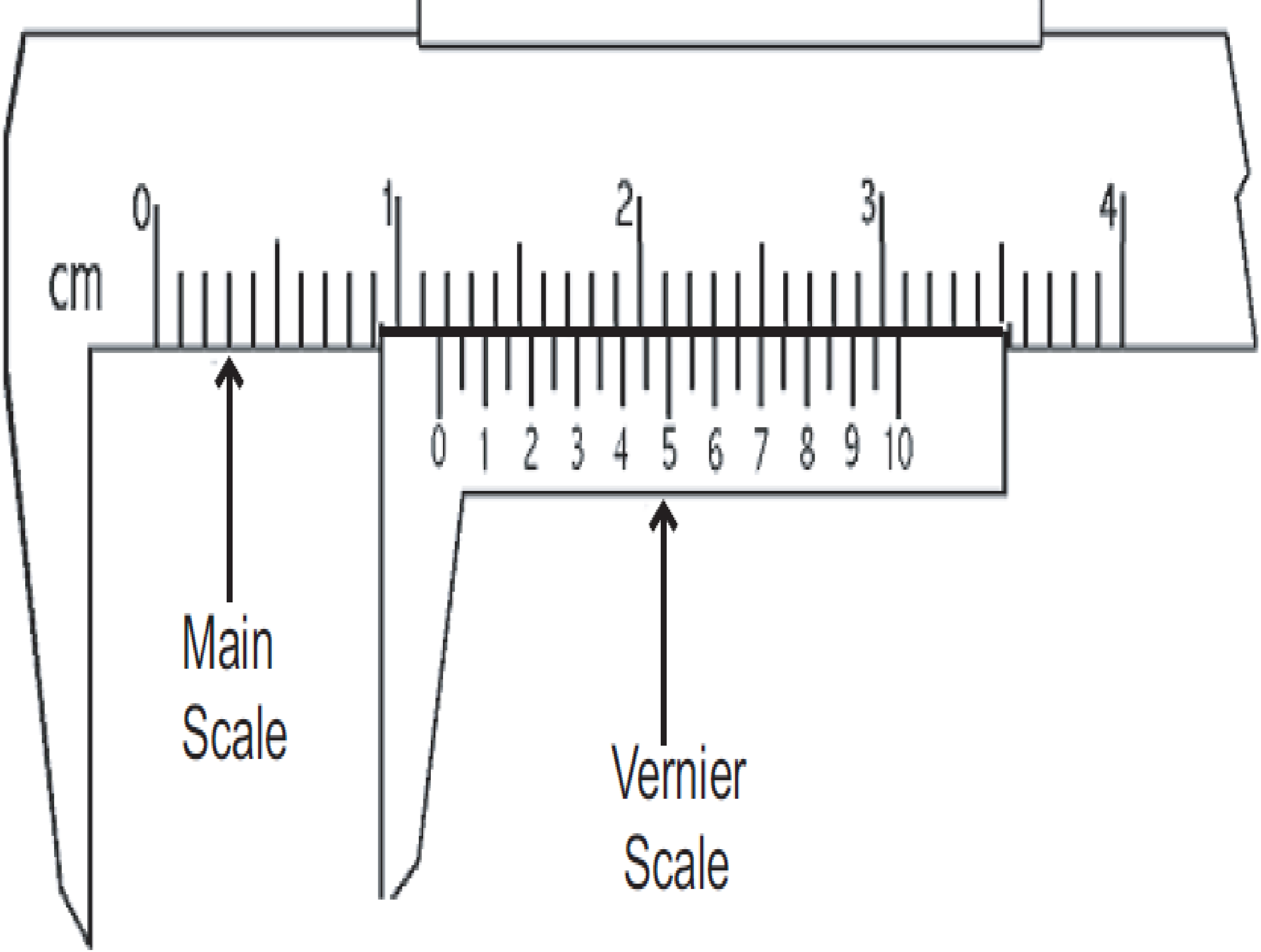
CLASSIFICATION OF THEODOLITES

B.(a)**Vernier Theodolite:** For reading the graduated circle if verniers are used ,the theodolite is called as a Vernier Theodolite

(b) Whereas, if a *micrometer* is provided to read the graduated circle the same is called as a **Micrometer Theodolite.**

(c)reading directly providing as digital num. if electronic distance measuring is attached with it then it's call total station

Vernier type theodolites are commonly used .


cm
0
1
2
3
4
0 1 2 3 4 5 6 7 8 9 10
Main Scale
Vernier Scale

25
20
0
15
10
5
Mahr
Micromar 40A
0-25

SIZE OF THEODOLITE

A theodolite is designated by diameter of the graduated circle on the lower plate.

The common sizes are *8cm to 12 cm* while *14 cm to 25 cm* instrument are used for *triangulation work*.

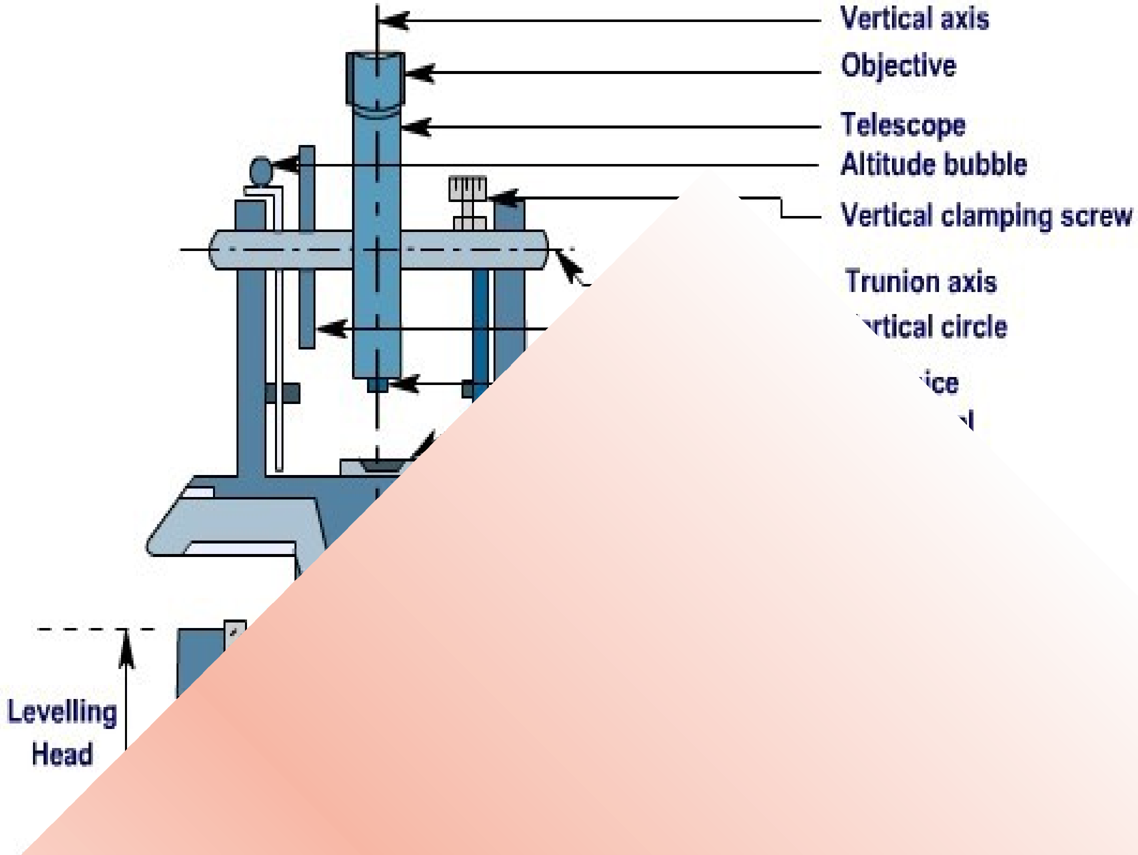
Greater accuracy is achieved with larger theodolites as they have bigger graduated circle with larger divisions hence used where the survey works require high degree of accuracy.

USES OF THEODOLITE

The Theodolite is a most accurate surveying instrument mainly used for :

- **Measuring horizontal and vertical angles.**
- **Locating points on a line.**
- **Prolonging survey lines.**
- **Finding difference of level.**
- **Setting out grades**
- **Ranging curves**
- **Tacheometric Survey**

TRANSIT VERNIER THEODOLITE


Vertical axis
Objective
Telescope
Altitude bubble
Vertical clamping screw
Trunion axis
rtical circle
ice
Levelling
Head

THEODOLITE SURVEYING

TRANSIT VERNIER THEODOLITE

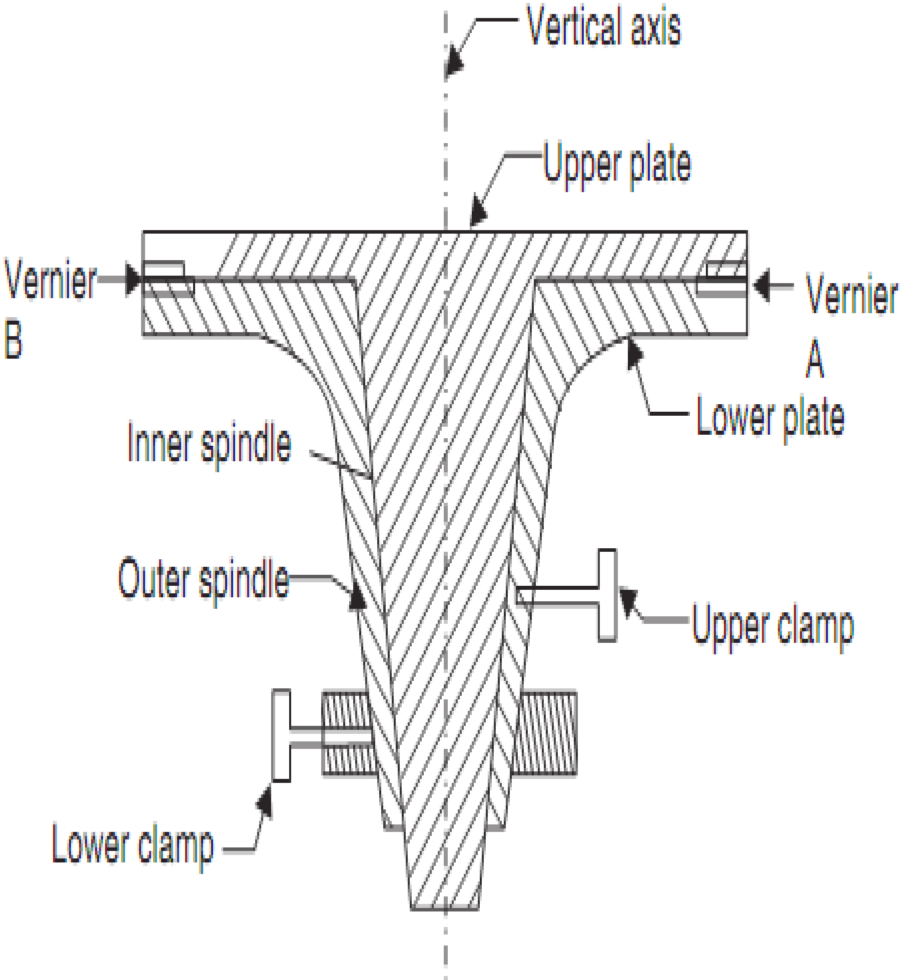

Vertical axis
Upper plate
Vernier B
Vernier A
Lower plate
Inner spindle
Outer spindle
Upper clamp
Lower clamp

Fig. Details if Upper & Lower Plates.

DESCRIPTION OF A TRANSIT VERNIER THEODOLITE

A Transit vernier theodolite essentially consist of the following :

1. Levelling Head.
2. Lower Circular Plate.
3. Upper Plate.
4. Telescope.
5. Vernier Scale.
6. T- Frame.
7. Plumb –bob.
8. Tripod Stand.

TERMS USED IN MANIPULATING A TRANSIT VERNIER THEODOLITE.

1. **Centering** : Centering means setting the theodolite exactly over an instrument-station so that its vertical axis lies immediately above the station- mark. It can be done by means of plumb bob suspended from a small hook attached to the vertical axis of the theodolite. The centre shifting arrangement if provided with the instrument helps in easy and rapid performance of the centring.

TERMS USED IN MANIPULATING A TRANSIT VERNIER THEODOLITE.

2. Transiting :

Transiting is also known as *plunging* or *reversing*. It is the process of turning the telescope about its horizontal axis through 180^0 in the vertical plane thus bringing it upside down and making it point , exactly in opposite direction.

TERMS USED IN MANIPULATING A TRANSIT VERNIER THEODOLITE.

3. Swinging the telescope

It means turning the telescope about its vertical axis in the horizontal plane.

A swing is called *right* or *left* according as the telescope is rotated clockwise or counter clockwise.

TERMS USED IN MANIPULATING A TRANSIT VERNIER THEODOLITE.

4. Face Left

If the vertical circle of the instrument is on the left side of the observer while taking a reading ,the position is called the *face left* and

the observation taken on the horizontal or vertical circle in this position, is known as the *face left observation*

TERMS USED IN MANIPULATING A TRANSIT VERNIER THEODOLITE.

5. Face Right

If the vertical circle of the instrument is on the right side of the observer while taking a reading ,the position is called the *face right* and

the observation taken on the horizontal or vertical circle in this position, is known as the *face right observation.*

TERMS USED IN MANIPULATING A TRANSIT VERNIER THEODOLITE.

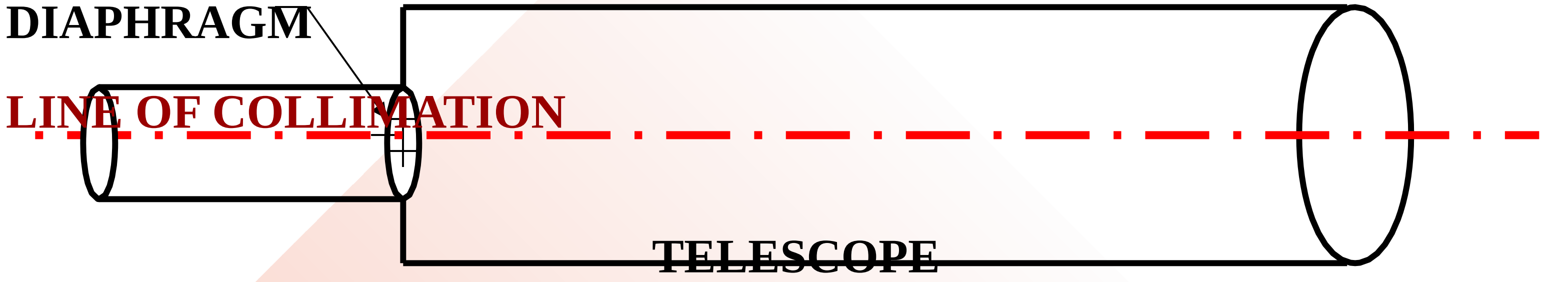
6. Changing Face

It is the operation of bringing the vertical circle to the right of the observer ,if originally it is to the left , and vice – versa.

It is done in two steps; Firstly revolve the telescope through 180^0 in a vertical plane and then rotate it through 180^0 in the horizontal plane i.e first transit the telescope and then swing it through 180^0.

TERMS USED IN MANIPULATING A TRANSIT VERNIER THEODOLITE.

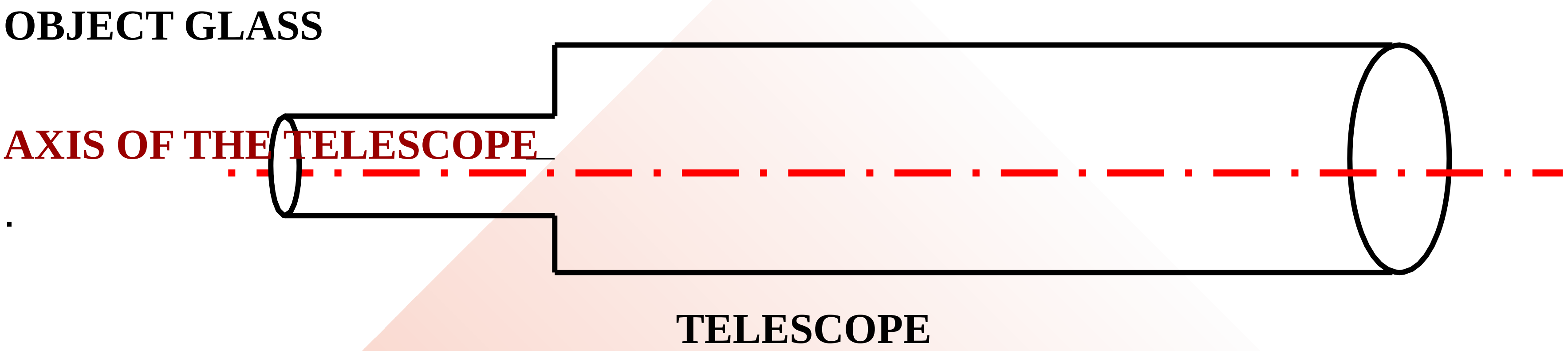
7. Line of Collimation


DIAPHRAGM
LINE OF COLLIMATION
TELESCOPE

It is also known as the line of sight .It is an imaginary line joining the intersection of the cross- hairs of the diaphragm to the optical centre of the object- glass and its continuation.

TERMS USED IN MANIPULATING A TRANSIT VERNIER THEODOLITE.

8. Axis of the telescope


OBJECT GLASS
AXIS OF THE TELESCOPE
TELESCOPE

It is also known an imaginary line joining the optical centre of the object- glass to the centre of eye piece.

TERMS USED IN MANIPULATING A TRANSIT VERNIER THEODOLITE.

9. Axis of the Level Tube

It is also called the bubble line.

It is a *straight* line *tangential* to the *longitudinal curve* of the level tube at the centre of the tube. It is horizontal when the bubble is in the centre.

TERMS USED IN MANIPULATING A TRANSIT VERNIER THEODOLITE.

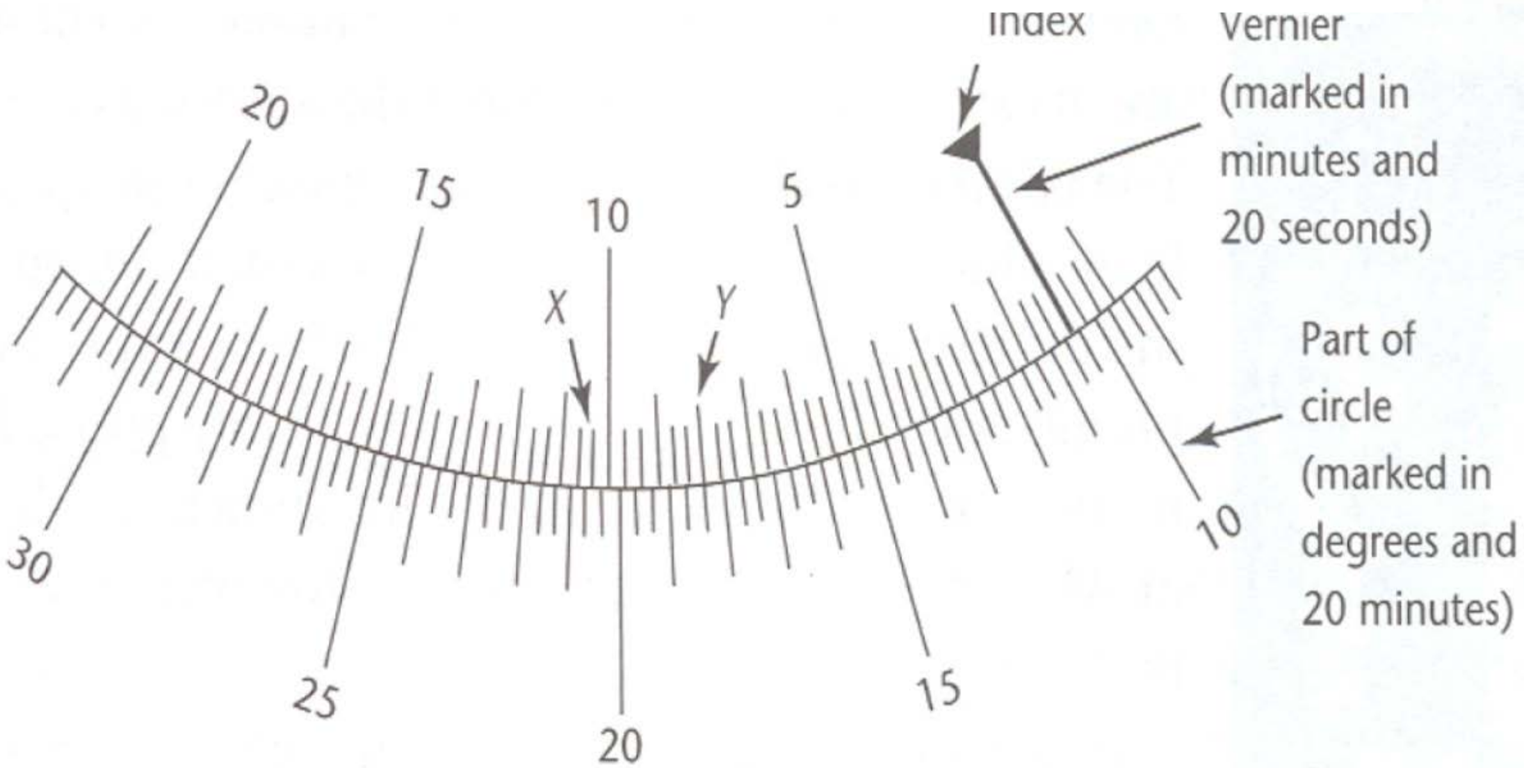
10.Vertical Axis

It is the axis about which the telescope can be rotated in the horizontal plane.

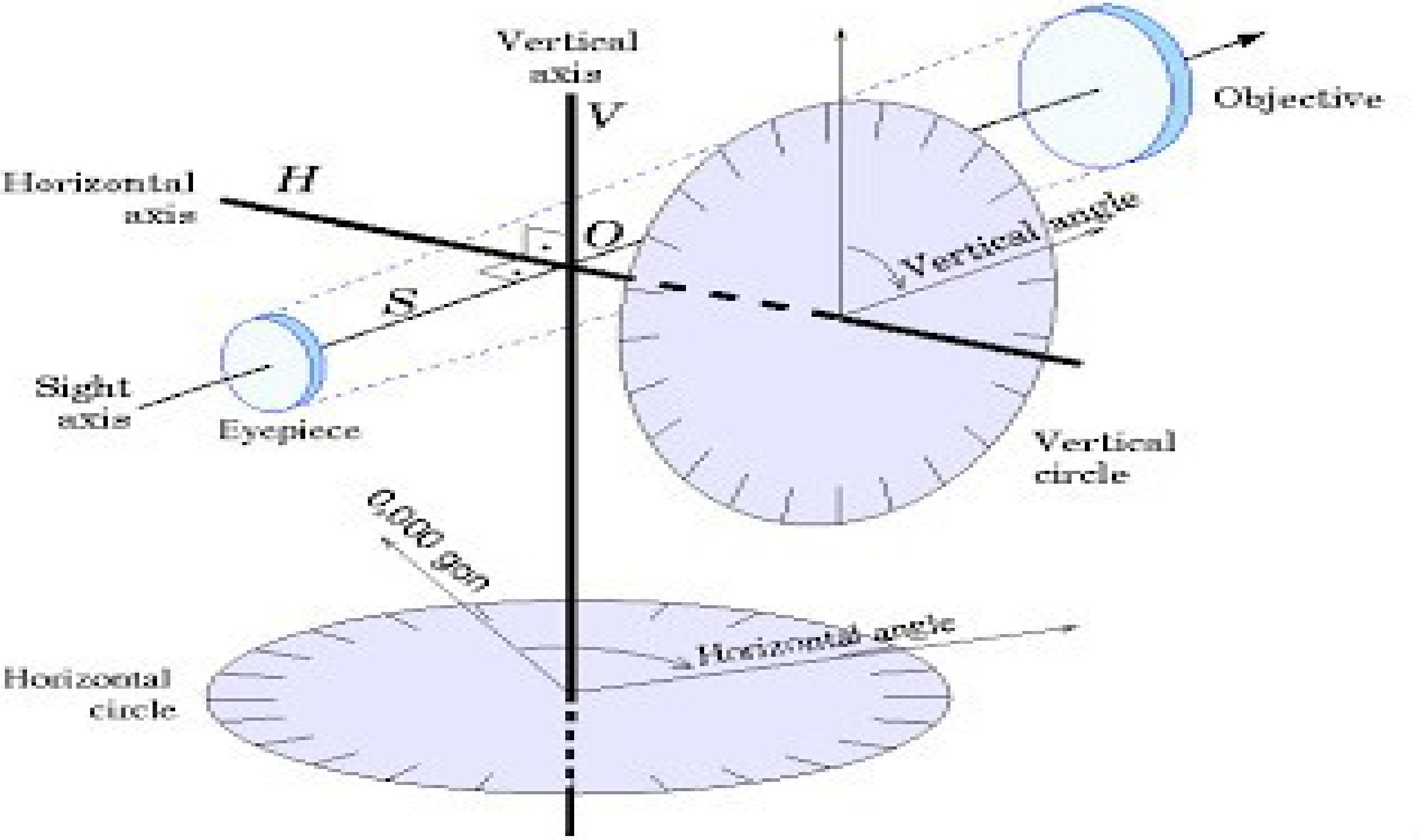
11.Horizontal Axis

It is the axis about which the telescope can be rotated in the vertical plane.
It is also called the *trunion axis.*

READING ON VERNIER SCALE


Index
Vernier
(marked in
minutes and
20 seconds)
Part of
circle
(marked in
degrees and
20 minutes)
20
15
10
5
X
Y
30
25
20
15
10

Fundamental axes of THEODOLITE & their relationship


Vertical axis
V
Objective
Horizontal axis
H
O
Vertical angle
S
Sight axis
Eyepiece
Vertical circle
0,000 gon
Horizontal angle
Horizontal circle

ADJUSTMENT OF A THEODOLITE

Temporary Adjustment

The temporary adjustments are made at each set up of the instrument before we start taking observations with the instrument. There are three temporary adjustments of a theodolite:-

i)Setting up & Centering.

ii)Levelling.

iii)Elimintion of parallax.

MEASUREMENT OF HORIZONTAL ANGLES:

There are three methods of measuring horizontal angles:-

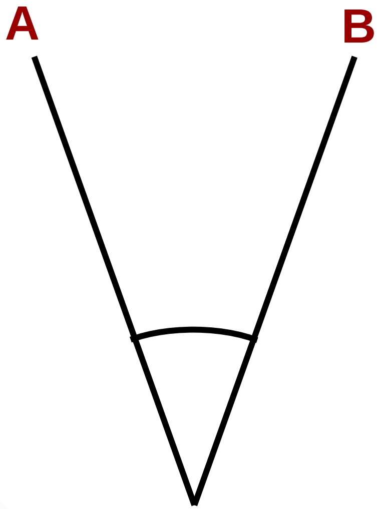
i)General Method.

ii)Repetition Method.

iii)Reiteration Method.

MEASUREMENT OF HORIZONTAL ANGLES:

a) general Method. To measure horizontal angle AOB:-



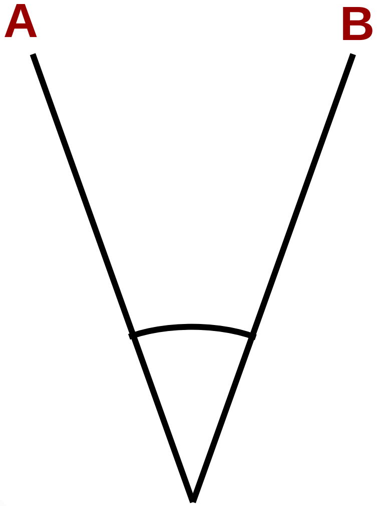
i. Set up the theodolite at station-point O and level it accurately.

ii. Loosen the lower clamp. Turn the instrument and direct the telescope Set the vernier A to the zero or 360° of the horizontal circle so do this, loosen the upper clamp and tum the upper plate until the zero of vernier A nearly coincides with the zero of the horizontal circle. Tighten the upper clamp and turn its tangent screw to bring the two zeros into exact coincidence.

iii. Loosen the lower clamp, Turn the instrument and direct the telescope approximately to the left hand object (A) by sighting over the top of the telescope. Tighten the lower clamp and bisect A exactly by turning the lower tangent screw. Bring the point A into exact coincidence with the point of intersection of the cross-hairs at diagram by using the vertical circle clamp and tangent screws.

MEASUREMENT OF HORIZONTAL ANGLES:

a) general Method. To measure horizontal angle AOB:-



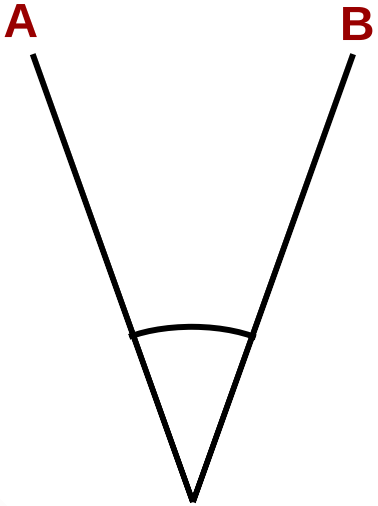
iv. Having sighted the object A, see whether the vernier A still reads zero. This is done to detect the error caused by turning the wrong tangent screw. Read the vernier B and record both vernier readings.

v. Loosen the upper clamp and turn the telescope clockwise until the line of sight is set nearly on the right hand object (B). Then tighten the upper clamp and by turning its tangent screw, bisect B exactly. In this operation, the lower clamp and its tangent screws should not be touched.

vi. Read both verniers: The reading of the vernier A which was initially set at zero gives the value of the angle AOB directly and that of the other vernier B by deducting 180°. The mean of the two vernier readings (after deducting 180° from the reading on vernier B gives the value of the required angle AOB.)

MEASUREMENT OF HORIZONTAL ANGLES:

a) general Method. To measure horizontal angle AOB:-



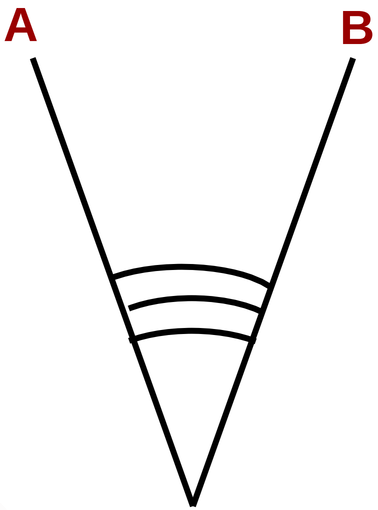
vii. Change the face of the instrument and repeat the whole process. The mean of the two vernier readings gives the second value of the angle ABC which should be approximately or exactly equal to the previous value.

viii. The mean of the two values of the angle AOB, one with the face left and the other with the face right, gives the required angle free from all instrumental errors.

MEASUREMENT OF HORIZONTAL ANGLES:

b) Repetition Method.

This method is used for very accurate work. In this method; the same angle is added several times mechanically and the correct value of the angle is obtained by dividing the accumulated reading by the number of repetitions. The number of repetitions made usually is six, three with the face left and three with the face right. In this way, angles can be measured to a finer degree of accuracy than that obtainable with the least count of the vernier.



i. Set up the theodolite at station -point O and level it accurately. (The face of the instrument should be left.)

ii. Set the vernier A to zero or 360° by using the upper clamp and its tangent screw. Then loosen the lower clamp, direct the telescope to the left hand object A, and bisect A exactly by using the lower clamp and its tangent screw.

MEASUREMENT OF HORIZONTAL ANGLES:

b) Repetition Method.

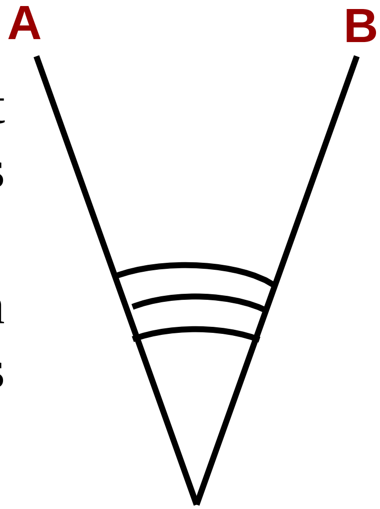
iii. Check the reading of the vernier A and see whether it still reads zero, and then read the other vernier B.

iv. Loosen the upper clamp, turn the telescope clock-wise and bisect the right hand object (B) exactly by using the upper clamp and its tangent screw.

v. Read both verniers- The object of reading the verniers is to obtain the approximate value of the angle. (Suppose the mean reading is 50°4′).

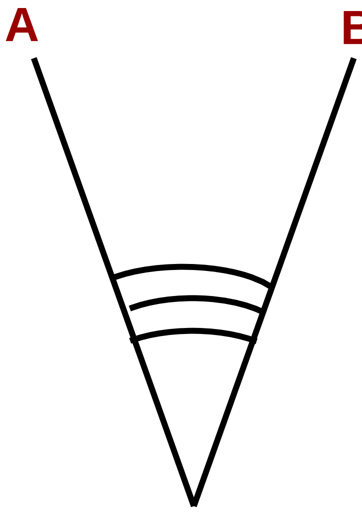
vi. Loosen the lower clamp and turn the telescope clock-wise until the object (A) is sighted again. Bisect A accurately using the lower tangent screw. Check the vernier readings which must be the same as before.

vii. Loosen the upper clamp, turn the telescope clock-wise and again sight towards B. Bisect B accurately by using the upper tangent screw. The verniers will now read twice the value of the angle (It should he approximately 100 °8′).



MEASUREMENT OF HORIZONTAL ANGLES:

b) Repetition Method.



viii. Repeat the process until the angle is repeated the required number of times (usually 3). Read both verniers. The final readings after n repetition should be approximately n x (50°4′). Divide the sum by the number of repetitions and the result thus obtained gives the correct value of the angle AOB.

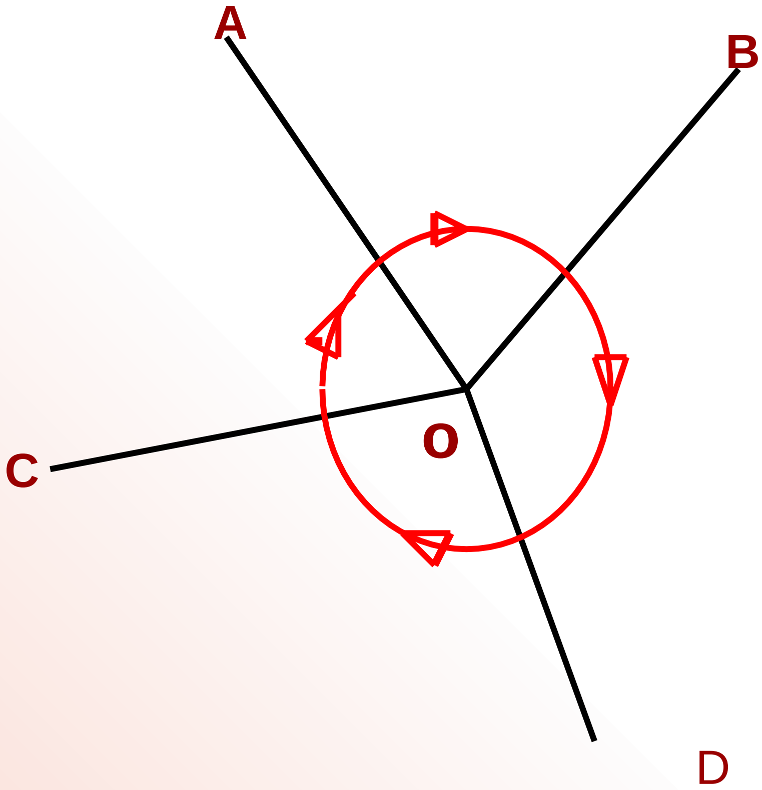
ix. Change the face of the instrument (now the face will be right). Repeat exactly in the same manner and find another value of the angle AOB.

x. The average of the two values of the angle thus obtained gives the required precise value of the angle (AOB).

MEASUREMENT OF HORIZONTAL ANGLES:

c) Reiteration Method.

Reiteration is another precise and comparatively less tedious method of measuring the horizontal angles. It is generally preferred when several angles are to be measured at a particular station. This method consists in measuring the several angles successively, and finally closing the horizon at the starting point. The final reading of the vernier A should be the same as its initial reading. If not, the discrepancy is equally distributed among all the measured angles. Suppose it is required to measure the angles **AOB**, **BOC** and **COD**.



Reiteration Method

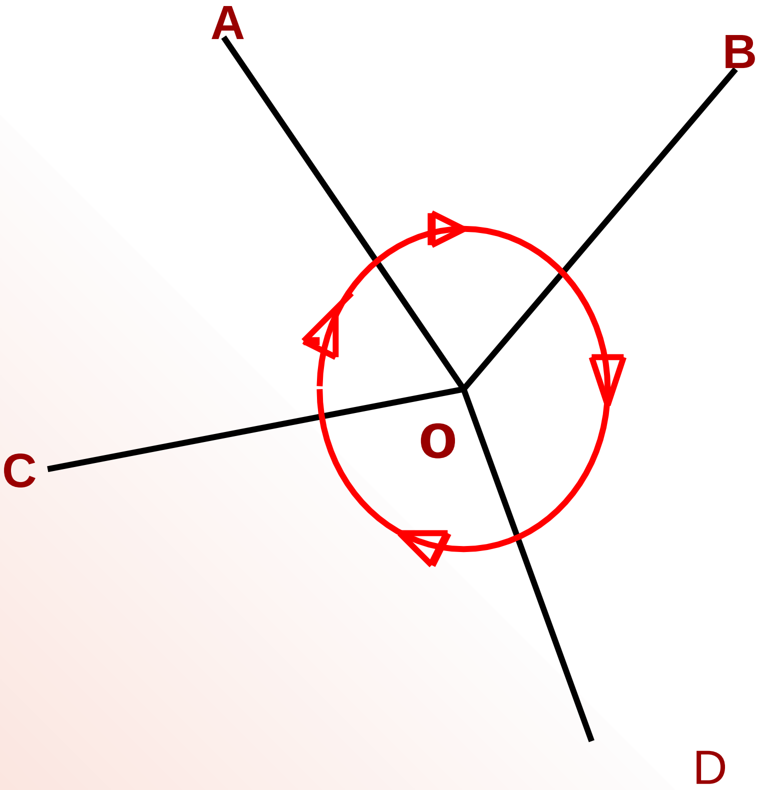
MEASUREMENT OF HORIZONTAL ANGLES:

c) Reiteration Method.

i. Set up the instrument over station point O and level it accurately.

ii. Set the vernier A to 0 or 360° by using the upper clamp and its tangent screw.

iii. Direct the telescope to some well-defined object (P) or say, the station point A, which is known as the 'Reference object'. Bisect it accurately by using the lower clamp and its tangent screw. Check the reading at vernier A which should still be 0 or 360° and note the reading at vernier B.



Reiteration Method

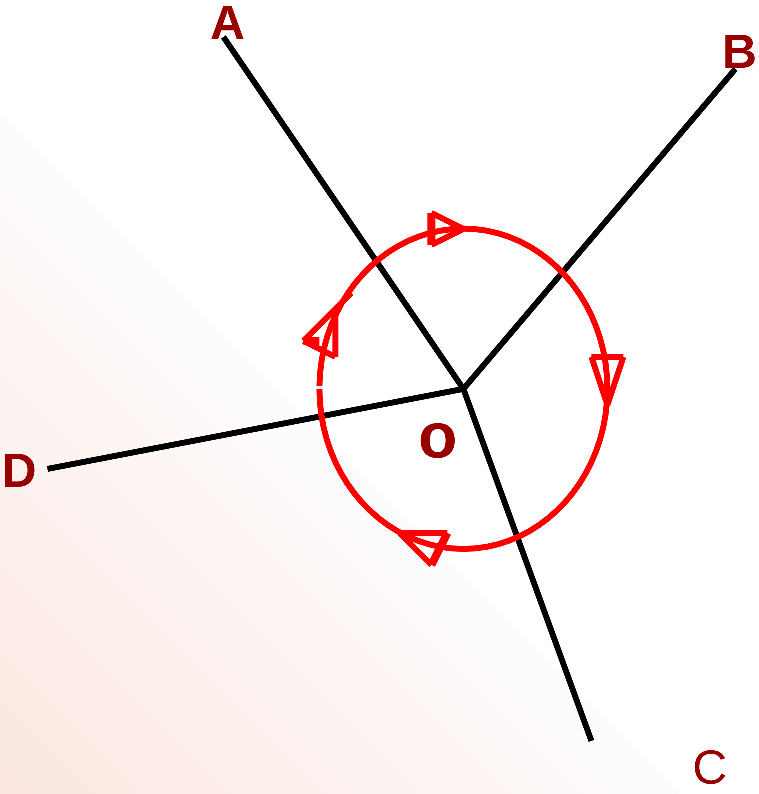
MEASUREMENT OF HORIZONTAL ANGLES:

c) Reiteration Method.

iv. Loosen the upper clamp and turn the telescope clockwise until the point B is exactly sighted by using the upper tangent screw. Read both verniers. The mean of the two vernier readings (after deducting 180° from the reading at vernier B) will give the value of the angle AOB.

v. Similarly bisect C and D successively, read both verniers at each bisection, find the values of the angles BOC and COD

vi. Finally, close the horizon by sighting towards the reference object (P) or the station-point A.



Reiteration Method

MEASUREMENT OF HORIZONTAL ANGLES:

c) Reiteration Method.

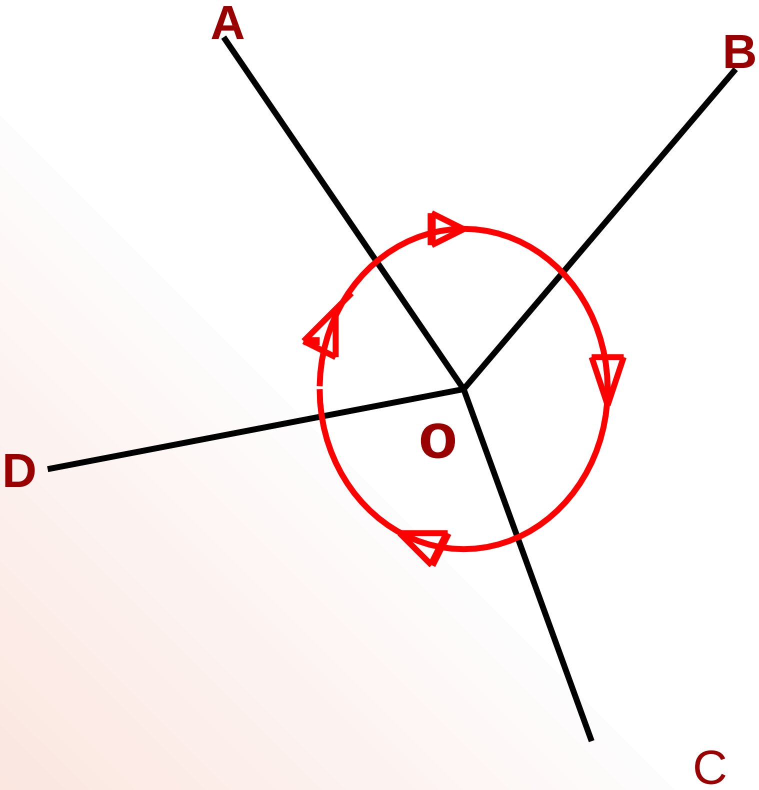
vii. The vernier A should now read 360°. If not, note down the error. This error occurs due to slip etc. Similarly bisect C and D successively, read both verniers at each bisection, find the values of the angles BOC and COD

viii. If the error is small, it is equally distributed among the several observed angles. If large, the readings should be discarded and a new set of readings be taken.

ix. Change the face of the instrument.

x. Set the vernier A to a reading other than 0°, say, 60° or 90°). This is done to avoid errors of graduation.

xi.



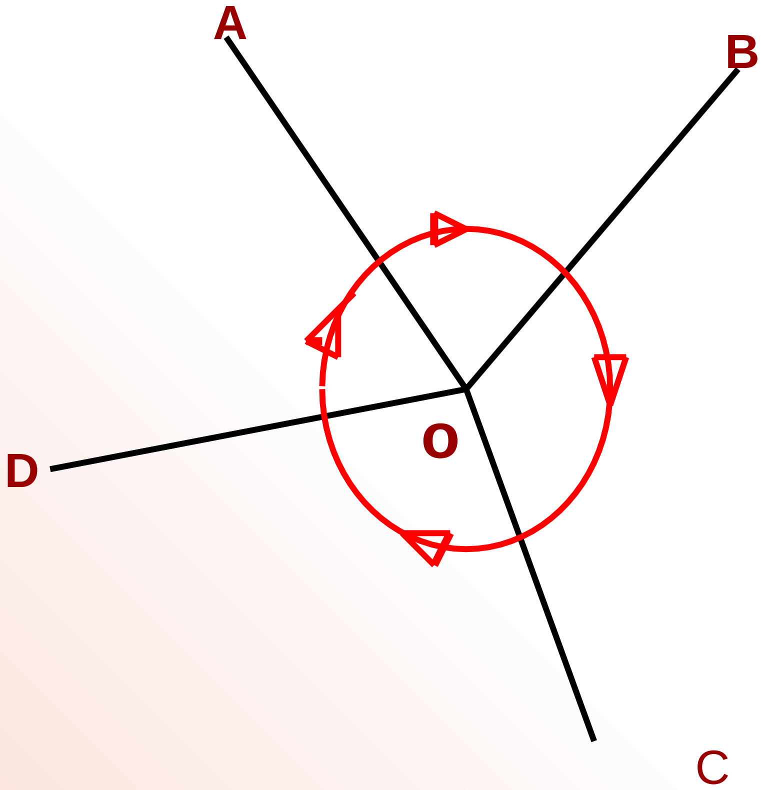
Reiteration Method

MEASUREMENT OF HORIZONTAL ANGLES:

c) Reiteration Method.

xi. Again measure the angles in the same manner by turning the telescope this time in the counter-clockwise direction to compensate or slip and errors due to twisting of the instrument.

xii. The mean of the two results for each angle is taken as its true value.



Reiteration Method

MEASUREMENT OF VERTICAL ANGLES:

Vertical Angle : **A vertical angle is an angle between th *inclined line of sight* and the *horizontal*. It may be a angle of *elevation* or *depression* according as the object above or below the horizontal plane.**

$\angle AOB = \alpha + \beta$

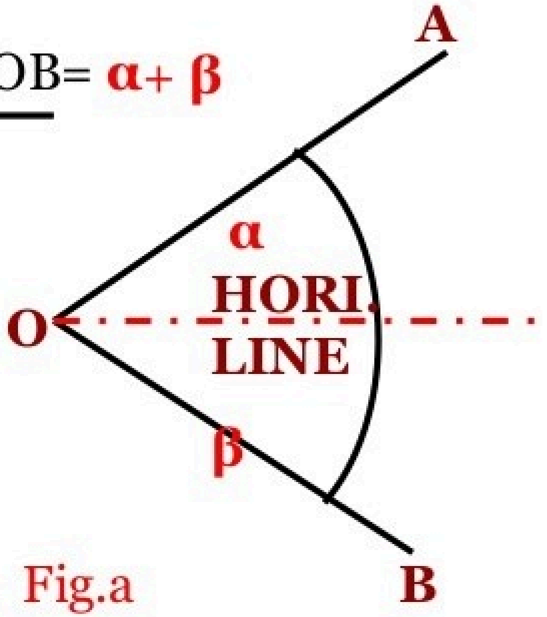

A
α
HORI.
LINE
O
β
B

Fig.a

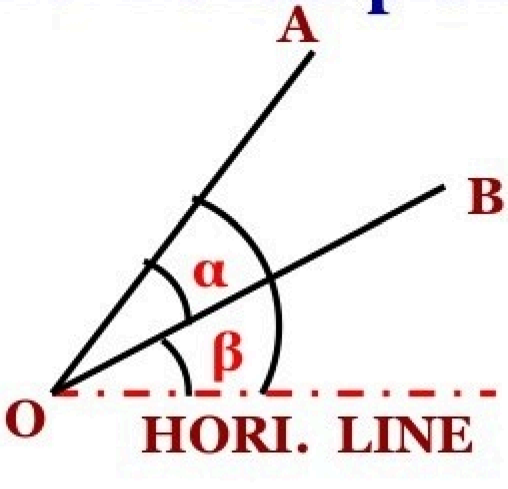

A
B
α
β
O
HORI. LINE

Fig. b

$\angle AOB = \alpha - \beta$

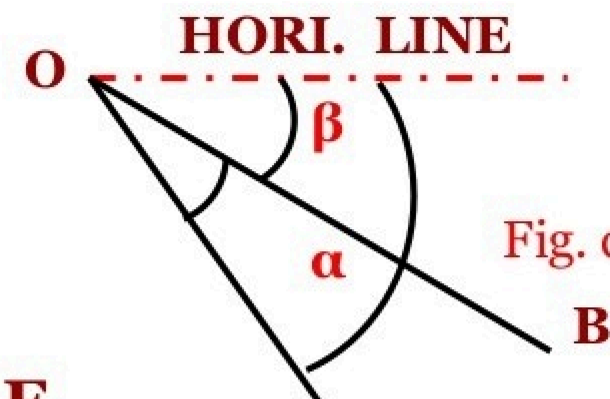

HORI. LINE
O
β
α
B

Fig. c

VERTICAL ANGLE

MEASUREMENT OF VERTICAL ANGLES:

To Measure the Vertical Angle of an object A at a station O:

(i) Set up the theodolite at station point O and level accurately with reference to the *altitude bubble*.

(ii) Set the zero of vertical vernier exactly to the zero of th vertical circle clamp and tangent screw.

(iii) Bring the bubble of the altitude level in the central position.

(iv) The line of sight is thus made horizontal and vernier sti reads zero.

(v) Loosen the vertical circle clamp screw and direct th telescope towards the object A and sight it exactly by usin the vertical circle tangent screw

MEASUREMENT OF VERTICAL ANGLES:

(v) Read both verniers on the vertical circle, The mean of the two vernier readings gives the value of the required angle.

(vi) Change the face of the instrument and repeat the process. The mean of of the two vernier readings gives the second value of the required angle.

(vii) The average of the two values of the angles thus obtained, is the required value of the angle free from instrumental errors.

MEASUREMENT OF VERTICAL ANGLES:

For measuring Vertical Angle between two points A &

i) Sight A as before , and take the mean of the two vern readings at the vertical circle. Let it be α

ii) Similarly, sight B and take the mean of the two vernier readings at the vertical circle. Let it be β

iii) The sum or difference of these readings will give the value of the vertical angle between A and B according as one of the poin is above and the other below the horizontal plane. or both points are on the same side of the horizontal plane Fig b & c

7.16 Draw sections A_1A_2, B_1B_2, and C_1C_2 from the contour map shown in Fig.7.22.

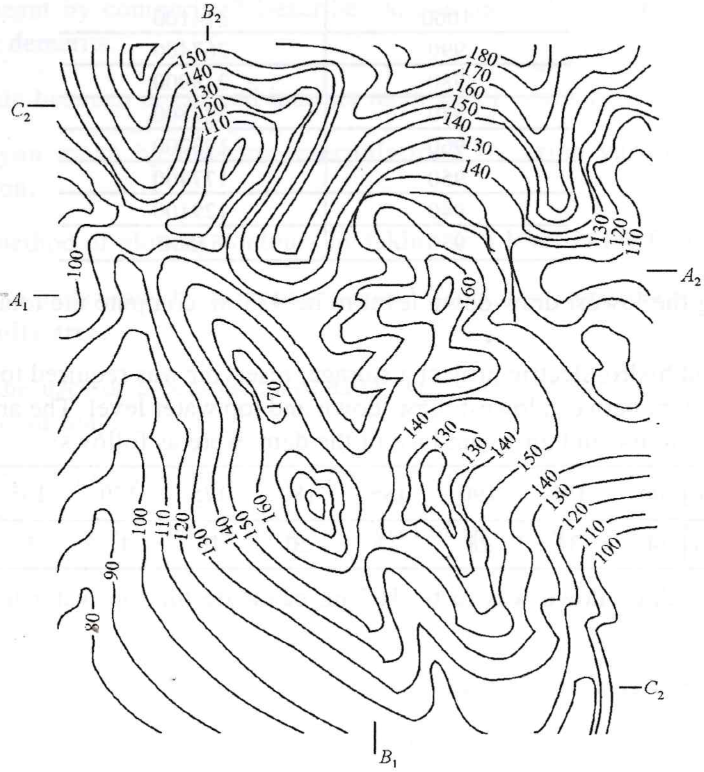

B_2
150
140
130
120
110
C_2
180
170
160
150
140
130
140
130
120
110
A_2
100
A_1
160
170
140
130
130
140
150
140
130
120
110
100
100
110
120
130
140
150
160
90
80
C_2
B_1

Fig. 7.22

7.17 Measure the areas contained within the contours having elevation of 150 m to 120 m on the upstream of the dam shown in Fig.7.23, using a planimeter and calculate the reservoir capacity.

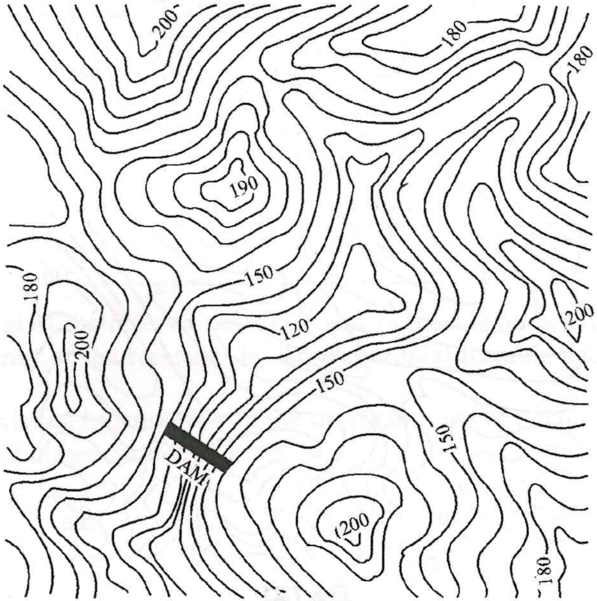

200
180
180
190
180
200
150
120
200
150
150
DAM
200
180

Fig. 7.23

8

TACHEOMETRY

8.1 GENERAL

Tacheometry is an indirect method by which horizontal distances and differences in elevation are determined using subtended intervals and angles observed with a transit or theodolite on a graduated scale. The word tacheometry is derived from the Greek words ταχυs (tachus) meaning swift, and μετρον (metron) meaning measure. As the name implies, the methods of tacheometric surveying are very rapid but are usually of a lower order of accuracy than is possible by conventional methods such as taping, differential levelling, etc. However, the results are very adequate for many purposes where taping is difficult. Tacheometry eliminates operation of taping, therefore, it is very useful for rough terrains such as ravines, river, valleys, steep slopes, broken grounds, stretches of water or swamps.

Stadia is a tacheometric form of distance measurement that relies on fixed angle intercept.

8.2 USES OF TACHEOMETRY

The tacheometric methods of surveying are used with advantage over the direct methods of measurement of horizontal distances and differences in elevations. Some of the uses are:

1. Preparation of topographic maps which require both elevations and horizontal distances.
2. Survey work in difficult terrain where direct methods are inconvenient.
3. Detail filling.
4. Reconnaissance surveys for highways, railways, etc.
5. Checking of already measured distances.
6. Hydrographic surveys, and
7. Establishing secondary control.

8.3 PRINCIPLE OF STADIA MEASUREMENTS

For the measurement of stadia distances, the reticle or diaphragm in the telescope of the theodolite (some levels) is equipped with two additional horizontal hairs (Fig. 8.1) in addition to the normal cross-hairs, one above and the other below the main horizontal hair. These additional horizontal hairs are known as *stadia hairs*. Let us assume that the telescope is external focussing type and the line of sight is horizontal. In Fig. 8.1, a and b represent the two stadia hairs, and A and B the corresponding points on the staff and c, is the middle horizontal hair having its corresponding point C on the staff.

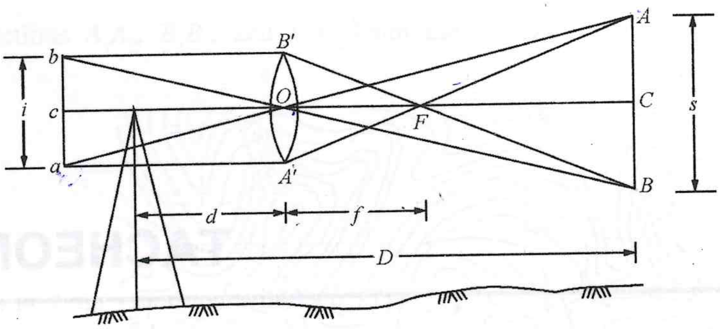


Fig. 8.1 Principle of transit stadia measurement

Let f = focal length of the object length

i = stadia hair interval ab

s = staff interval AB

D = horizontal distance of the staff from the instrument axis

d = distance of the optical centre of the object glass from the instrument axis.

From similar $\Delta^s A'B'F$ and ABF, we get

$$\frac{CF}{OF} = \frac{AB}{A'B'}$$

or
$$CF = OF\frac{AB}{A'B'}$$

We have $A'B' = ab$

$$CF = OF\frac{AB}{ab}$$

Substituting the value of AB, OF, and ab, we have

$$CF = \frac{f}{i}s$$

But $D = CF + f + d$

or
$$= \frac{f}{i}s + (f + d) \qquad \text{...(8.1)}$$

Substituting k for $\frac{f}{i}$ and c for $(f + d)$, the Eq. (8.1) becomes

$$D = ks + c \qquad \text{...(8.2)}$$

In Eq. (8.2), k and c are known as the *tacheometric constants*.

To make the computations simple, the stadia hairs are positioned in the reticle so that if a levelling staff or rod is held 100 m away from the theodolite, the difference between the upper and lower stadia hair readings on the staff, would be exactly 1.00 m for an internal focussing instruments. Figure 8.2 shows reticles with different patterns of stadia hairs used in theodolite.

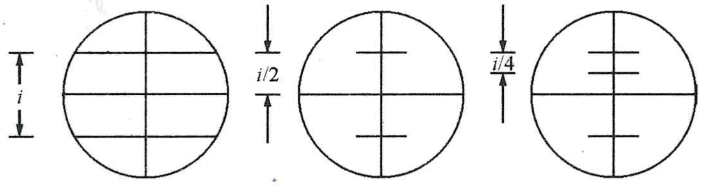


Fig. 8.2 Reticles with different patterns of stadia hairs

Generally, the values of the multiplying constant k and the additive constant c are kept equal to 100 and 0 (zero), respectively. With these values of k and c, the distance can be determined simply by sighting a staff with the telescope horizontal and determining the staff interval (Fig 8.3). The staff interval is then multiplied by 100 to get the horizontal distance, i.e.,

$$D = 100\,s \qquad \ldots(8.3)$$

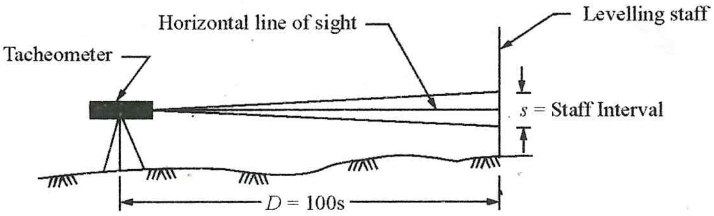

Horizontal line of sight
Levelling staff
Tacheometer
s = Staff Interval
D = 100s

Fig. 8.3 Horizontal distance by stadia method

Elevations are obtained by stadia by determining the height of the instrument (H.I.) and middle hair reading on the staff. The computation of the elevation of B, if the elevation of the instrument station A is known, is illustrated in (Fig. 8.4).

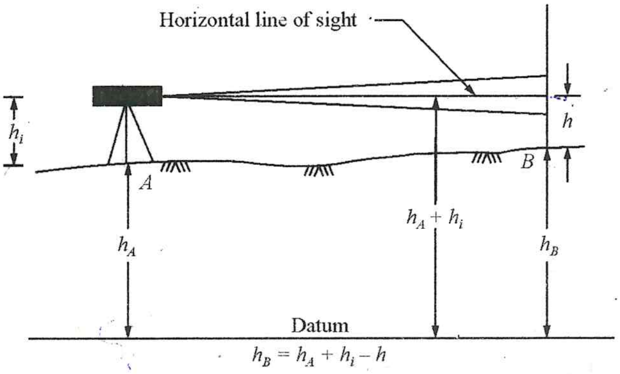

Horizontal line of sight
h_i
h
A
B
$h_A + h_i$
h_A
h_B
Datum
$h_B = h_A + h_i - h$

Fig. 8.4 Elevation by stadia method

Inclined stadia measurements

The stadia method is particularly well suited for the inclined sights required by rolling topography.

When the telescope is inclined up or down (Fig. 8.5) the computations must account for the inclined sights. For an internal focussing telescope having $c = 0$ and considering the angles $CM'M$ and $CN'N$ as right angles,

$$CM' = CM \cos\alpha$$

or $$\frac{s'}{2} = \frac{s}{2}\cos\alpha$$

or $$s' = s\cos\alpha \qquad \text{...(8.4)}$$

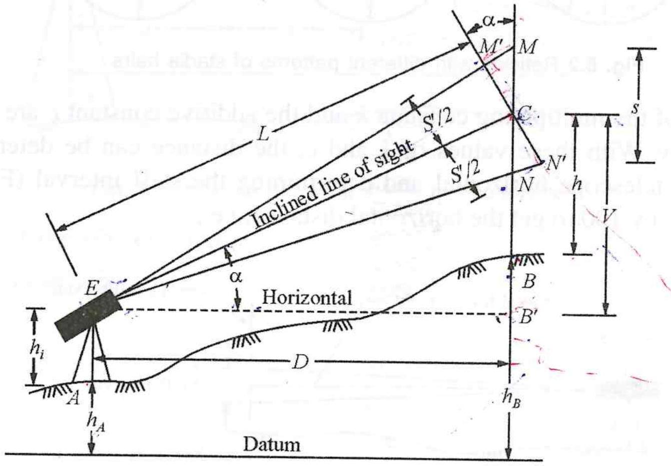


Fig. 8.5 Inclined stadia measurement ($c = 0$)

If $M'N'$ is the staff interval s', then the slope distance L from Eq. (8.1) is given by

$$L = ks'$$

$$= ks\cos\alpha \qquad \text{...(8.5)}$$

and the horizontal distance by

$$D = L\cos\alpha$$

From Eq. (8.5), we have

$$D = ks\cos^2\alpha \qquad \text{...(8.6)}$$

The vertical distance is given by

$$CB' = V = D\tan\alpha$$

$$= ks\cos^2\alpha\,\frac{\sin\alpha}{\cos\alpha} = ks\sin\alpha\cos\alpha$$

$$= \frac{1}{2}ks\sin 2\alpha \qquad \text{...(8.7)}$$

The elevation h_B of B is equal to the elevation of instrument axis + V – middle-hair reading, *i.e.*,

$$h_B = h_A + h_i + V - h \qquad \text{...(8.8)}$$

8.4 TACHEOMETER AND STADIA RODS

A *tacheometer* is a theodolite or transit, fitted with a stadia reticle or diaphragm discussed in Sec.8.3.

For short distances, say upto 100 m or so, an ordinary vernier theodolite with stadia diaphragm may be used. For long sights, tacheometers with high magnifying power are used for accurate readings. The aperture of the objective of such tacheometers is quite large so as to have a bright image. The multiplying constant k of a tacheometer is mostly 100. The additive constant is either zero or negligible. The tacheometers having external focussing telescopes, are provided with an anallactic lens to make the additive constant equal to zero.

Stadia rods are the levelling staffs having uniform graduations. For short distances ordinary levelling staff with 5 mm graduations can be used. For long distances a stadia rod, 3 to 5 m long, 50 to 150 mm wide having simple and bold graduations as shown in Fig. 8.6, for taking readings from long distances is used.

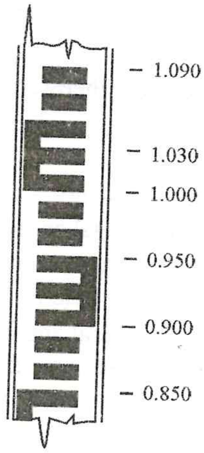


Fig. 8.6 Stadia rod

8.5 BASIC SYSTEMS OF TACHEOMETRIC MEASUREMENTS

There are two following basic systems of tacheometric measurements:

1. Stadia systems
   (a) Fixed-hair method
   (b) Movable-hair method.
2. Non-stadia systems
   (a) Tangential system
   (b) Subtense bar system.

Stadia systems

The principle of stadia system has already been discussed in Sec. 8.3.

The two methods of stadia system are discussed below.

Fixed-hair method

In this method, stadia hairs are kept at fixed interval and the staff interval or intercept on the levelling staff varies, depending upon the horizontal distance between the instrument station and the staff. This is the most common method in tacheometry and the name stadia-hair method generally refers to this method, unless mentioned otherwise.

Movable-hair method

In this method, the staff interval is kept constant by changing the distance between the stadia hairs. Targets on the staff at a known fixed distance apart are fixed and the stadia hairs are adjusted such that the upper hair bisects the upper target and the lower hair bisects the lower one. Instruments used for this system of tacheometry, have provision for the measurement of the variable interval between the stadia hairs.

As it is inconvenient to measure the stadia interval accurately, the movable-hair method is rarely used.

Non-stadia systems

In this system, the stadia hairs are not required, therefore, the telescopes not provided with a stadia diaphragm can be used.

Tangential system

In the tangential system, readings on a staff are taken against the horizontal cross hair. To measure the staff intercept, two pointings of the telescope are necessary. Readings to full metre values on the staff are generally taken to avoid decimal parts for ease in computations.

The method is generally not adopted as two vertical angles are required to be measured for one single observation. It involves more computations and less speed.

Subtense bar system

In this system, a bar of fixed length, called a subtense bar is used (*cf.*, Sec 8.9). The bar is placed in horizontal position, and the angle subtended at the instrument station by the two targets of the subtense bar is measured. The horizontal distance between the subtense bar and the instrument is computed from the known distance between the targets and the measured subtended horizontal angle.

ILLUSTRATIVE EXAMPLES

Example 8.1

The following readings were taken with a transit fitted with stadia hairs. The line of sight was horizontal and the staff was held vertical.

	Reading on staff (m)
Top hair	0.875
Middle hair	1.340
Bottom hair	1.805

If the tacheometric constants k and c have the value as 100 and 0.20 m, respectively, what is the horizontal distance between the staff and the instrument?

Solution:

From Eq. (8.2), we get

$$D = ks + c$$

$$s = 1.805 - 0.875 = 0.930 \text{ m}$$

$$D = 100 \times 0.930 + 0.20 = \mathbf{93.200\ m}.$$

Example 8.2

Taking the data from Example 8.1, determine the R.L. of the staff station if the R.L. of the instrument station is 201.20 m and the height of the instrument above the ground is 1.51 m.

Solution:

From Fig. 8.4, we have

$$h_B = h_A + h_i + V - h$$

$$= 201.20 + 1.51 - 1.340 = \mathbf{201.37\ m.}$$

Example 8.3

To determine the distance between two points *P* and *Q*, and the R.L. of *Q*, the following observations were made:

Height of tacheometer at $P = 1.480$ m

Vertical angle at $P = +5°20'$

Staff readings (with staff vertical) = 0.545, 0.905, 1.265

R.L. of $P = 150.000$ m

$k = 100.00$

$c = 0.00$ m.

Solution (Fig. 8.7):

From Eq. (8.6), we have

$$D = ks\cos^2\alpha$$ ✓ 1mk

We have

$$s = 1.265 - 0.545 = 0.720\ \text{m}$$ — 2mks

$$\alpha = +5°20'$$

$$D = 100 \times 0.720 \times \cos^2 5°20' = \mathbf{71.378\ m.}$$ 2 mks

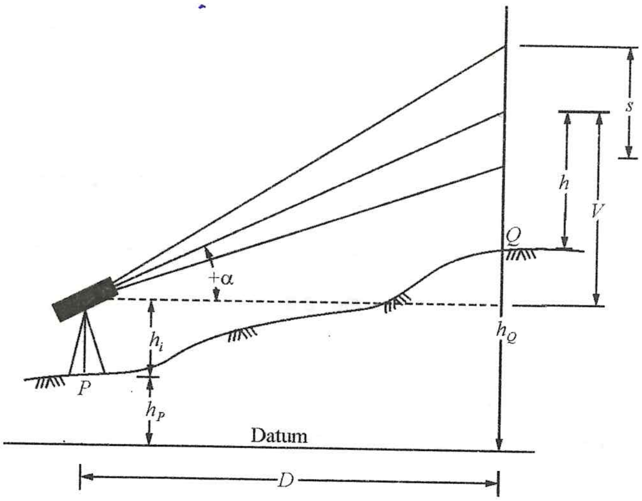


Fig. 8.7

From Eq. (8.8), we have

R.L. of $Q = h_P + h_i + V - h$ — 1 MK

We have $h_i = 1.480$ m

$h = 0.h_A905$ m

From Eq. (8.7), we get

$$V = \frac{1}{2} ks \sin 2\alpha$$ — 1 MK

$$= \frac{1}{2} \times 100 \times 0.720 \times \sin (2 \times 5°20') = 6.663 \text{ m}$$

R.L. of $Q = 150.000 + 1.480 + 6.663 - 0.905 = \mathbf{157.238}$ **m.** - 3 MK

Example 8.4

Determine the distance between the points P and Q from the following data :

R.L. of the tacheometer axis at $P = 185.300$ m.

Vertical angle at $P = -4°35'$

Staff readings at $Q = 1.440, 0.900, 0.360$

Also determine the R.L. of Q if the staff at Q was held vertical, and $k = 100$ and $c = 0.00$ m.

Solution (Fig. 8.8):

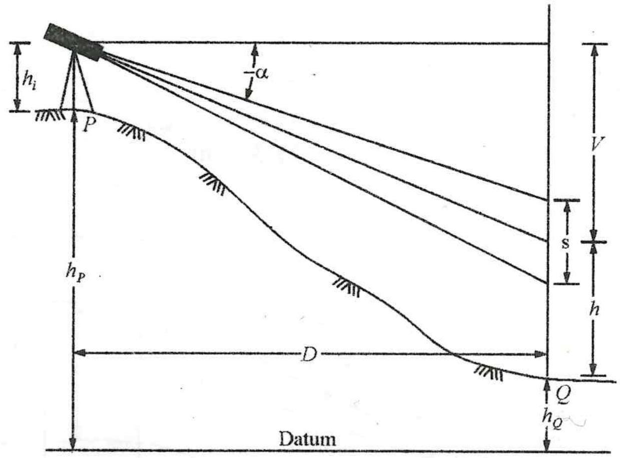


Fig. 8.8

We have $h_P + h_i = 185.300$ m

$h = 0.900$ m

$s = 1.440 - 0.360 = 1.080$ m

$\alpha = -4°35'$

From Eq. (8.6), we get

$$D = 100 \times 1.080 \times \cos^2 4°35' = 107.310 \text{ m}$$

From Eq. (8.7), we get

$$V = \frac{1}{2} \times 100 \times 1.080 \times \sin (2 \times 4°35')$$

$$= 8.603 \text{ m}$$

R.L. of Q $$h_Q = h_P + h_i - V - h$$

$$= 185.300 - 8.603 - 0.900$$

$$= \mathbf{175.797 \text{ m}}.$$

Example 8.5

To determine the distance between two stations A and B, a tacheometer was set up at a point P on the line AB and the following observations were made:

(*a*) When the staff was held at A

Staff readings = 2.225, 2.605, 2.985

Vertical angle = + 8°24′

(*b*) When the staff was held at B

Staff readings = 1.640, 1.920, 2.200

Vertical angle = – 1°06′

Also determine the R.L. of B if the R.L. of A is 315.673 m (k = 100.00 and c = 0.00).

Solution (Fig. 8.9):

We have $s_1 = 2.985 - 2.225 = 0.760$ m

$s_2 = 2.200 - 1.640 = 0.560$ m

$h_1 = 2.605$ m

$h_2 = 1.920$ m

$\alpha_1 = +\,8°24'$

$\alpha_2 = -1°06'$

$D_1 = 100 \times 0.760 \times \cos^2 8°24' = 74.378$ m

$D_2 = 100 \times 0.560 \times \cos^2 1°06' = 55.979$ m

Distance $AB = D_1 + D_2 = 74.378 + 55.979 = \mathbf{130.357}$ **m**.

also $V_1 = 1/2 \times 100 \times 0.760 \times \sin (2 \times 8°24') = 10.983$ m

$V_2 = 1/2 \times 100 \times 0.560 \times \sin (2 \times 1°06') = 1.075$ m

The R.L. of the instrument axis is

$$= h_A + h_1 - V_1$$

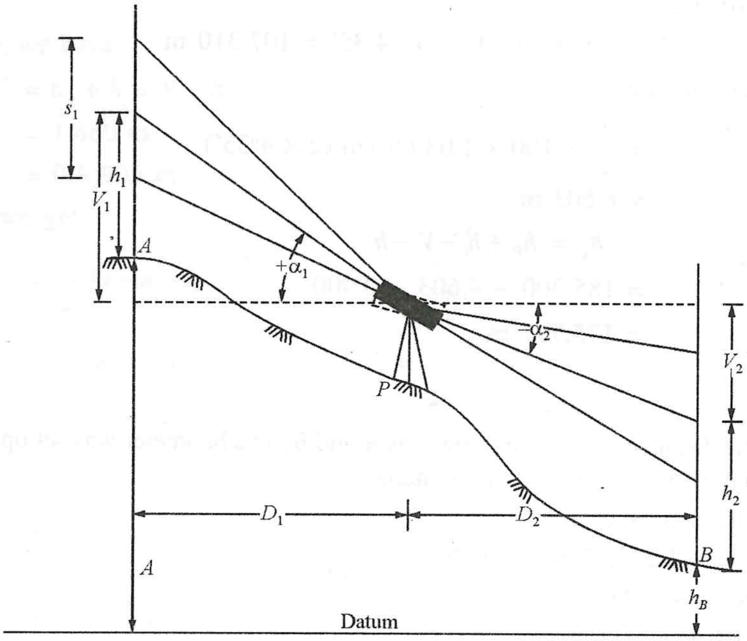


Fig. 8.9

The R.L. of *B* is

$$
\begin{aligned}
h_B &= \text{R.L. of the instrument axis} - V_2 - h_2 \\
&= h_A + h_1 - V_1 - V_2 - h_2 \\
&= 315.673 + 2.605 - 10.983 - 1.075 - 1.920 \\
&= \mathbf{304.300\ m}.
\end{aligned}
$$

Example 8.6

The vertical angles to vanes fixed at 1 m and 3 m above the foot of the staff held vertically at a station *A*, were + 2°15′ and + 5°50′, respectively. Determine the horizontal distance and reduced level of *A* if the height of the instrument axis is 235.665 m above datum.

Solution (Fig. 8.10):

We have $h_1 = 1$ m

$h_2 = 3$ m

$\alpha_1 = +2°15'$

$\alpha_2 = +5°50'$

$V_1 = D \tan \alpha_1$

$V_2 = D \tan \alpha_2$

but $V_2 - V_1 = h_2 - h_1 = 3 - 1 = 2$ m

$D (\cos \alpha_2 - \cos \alpha_1) = 2$

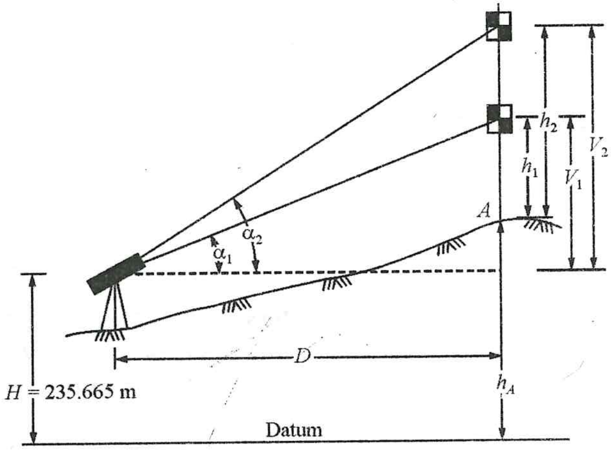


Fig. 8.10

or $$D = \frac{2}{\tan 5°50' - \tan\ 2°15'} = 31.810\ \text{m}$$

Thus,

$$V_1 = 31.810 \times \tan 2°\,15'$$
$$= 1.250\ \text{m}$$

The R.L. of A, $h_A = H + V_1 - h_1$

$$= 235.665 + 1.250 - 1.000$$
$$= \mathbf{235.915\ m}.$$

8.6 DETERMINATION OF TACHEOMETRIC CONSTANTS

The following two field methods are most common for determining the values of the tacheometric constants k and c.

Method-I

In this method, c is determined by directly measuring f and d (Fig. 8.1), and k by field observations.

(*a*) Measure the distance between the object glass and the plane of the cross-hairs with an accurate scale. This distance is approximately equal to f. Then measure accurately the distance between the object glass and the vertical axis of the instrument. This is equal to d. The value $(f + d)$ is equal to c.

(*b*) Set up the instrument at any point P and level it. Sight any distant object Q and focus the instrument using focussing screw. Measure the distance PQ. Read the staff interval s for top and bottom stadia hairs for horizontal line of sight.

Let PQ is equal to D. From Eq. (8.2), we get

$$D = ks + c$$

2

SURVEY MEASUREMENTS, ERRORS AND PROPAGATION OF ERRORS

2.1 GENERAL

The purpose of surveying is essentially to locate the positions of points on or near the surface of earth by making field measurements of distances and angles. Often, these basic field measurements are used to calculate other quantities of direct interest such as areas and volumes. The measurements always contain errors due to human limitations, imperfection in instruments, environmental changes, or carelessness on the part of the observer. Since the calculated quantities are evaluated through relationships with the measured quantities, the errors in measured quantities get propagated into the calculated quantities.

The surveyors should understand the different types of errors, their sources and how they propagate. He should know how to minimise the errors while taking the measurements. The errors which still creep in should be eliminated or their effects should be minimised as far as possible. He should also be able to find the errors in the calculated quantities.

2.2 RELIABILITY OF MEASUREMENTS

Several terms are used to express the reliability of measurements. The two most common are

1. Precision
2. Accuracy.

Precision is the degree of closeness or conformity of repeated measurements of the same quantity to each other whereas the *accuracy* is the degree of closeness or conformity of a measurement to its true value.

It is perfectly feasible to have a data set that is highly accurate but not very precise, and vice versa. The difference between accuracy and precision is best illustrated in Fig. 2.1, that is results of a shooting competition between four contestants *A*, *B*, *C*, and *D*. The shooting results indicate that

(*i*) Contestant *A* is accurate but not precise,

(*ii*) Contestant *B* is inaccurate but not precise,

(*iii*) Contestant *C* is neither accurate nor precise, and

(*iv*) Contestant *D*, the winner, is both accurate and precise.

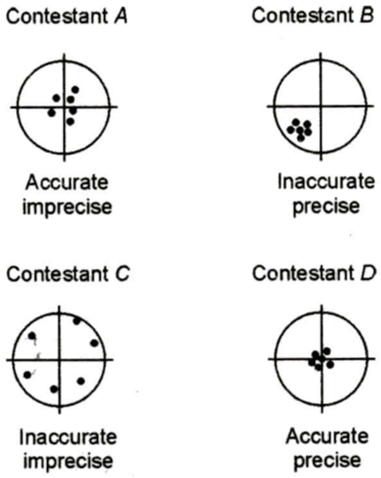


Fig. 2.1 Accuracy versus precision

Significant Figures

The number of digits or *significant figures* is an indication of the accuracy of a measurement. Only these digits in a measured number have significance. The number of significant figures in a numerical quantity equals "the number of digits in the quantity less all zero digits that are used to fix the position of the decimal point." The following examples make this clear.

Two significant figures : 32, 3.2, 0.32, 0.0032, 0.030
Three significant figures : 131, 13.1, 0.000131, 0.0320
Four significant figures : 3456, 34.56, 0.00003456, 32.00

The significant figures are very important from the following point of views.

(*i*) If large number of operations are required in computations, unnecessary digits retained after significant figures will take extra computation time resulting into wastage of time and money.

(*ii*) On the other hand, if one or two digits extra than required ones are not used in computations, the final computed values of a quantity are badly affected due to rounding off errors.

Rounding Off Numbers

Dropping one or more digits so that only significant ones or those that are necessary and sufficient for subsequent calculations, is known as *rounding off number*. If more digits have been dropped than the required ones, the result will have rounding off errors.

For example $\frac{1}{7}$ is equal to 0.143.........is in error by 0.0001428...............

The rules given below should be followed in rounding off numbers:

Rule 1: If the digit to be dropped is less than 5, it is ignored. For example, 38.274 is rounded off to 38.27.

Rule 2: If the digit to be dropped is greater than 5, increase the preceding digit by one. For example, 38.276 is rounded off to 38.28.

Rule 3: If the digit to be dropped is 5, the most common practice, which produces balanced result, is to use nearest even number for the previous digit, *e.g.*, 38.275 will become 38.28 and 38.285 will also be taken as 38.28.

2.3 CONCEPT OF ERRORS IN MEASUREMENTS

Different conditions discussed in Sec.2.1 under which the measurements are made, cause variations in measurements, and therefore, no measured quantity is completely determinable. A fixed value of a quantity concieved as its *true value*, can never be measured, as some errors will always creep in during its measurement, and what we get is nothing more than an *estimate* of the true value.

The difference between the measured quantity x and its true value τ is known as *error* ε, *i.e.*,

$$\varepsilon = x - \tau \qquad \text{...(2.1)}$$

Mathematically the measurement or observation is a variable, and study of observational errors and their behaviour is essentially the study of observations itself. *Theory of errors* deals with the study of observational errors.

Since the true value of a measurement cannot be determined, the exact value of ε can never be found out. However, if we can obtain a good estimate $\hat{x}$ of τ, then $\hat{x}$ can be used as a reference to express the variation in observed values x. Therefore, if we define υ as *residual*, then

$$\upsilon = \hat{x} - x \qquad \text{...(2.2)}$$

The residuals express the variations in the measurements. The quantity ($\hat{e}$) equal to ($-\upsilon$) is called *correction* to the measured quantity or *residual error*, *i.e.*,

$$(-\hat{e}) = \hat{x} - x$$

or Residual $= \hat{x} - x$

and residual error $= x - \hat{x}$.

2.4 TYPES OF ERRORS

Errors have been traditionally classified into the following three types:

1. Gross errors (blunders or mistakes),
2. Systematic errors, and
3. Random or accidental errors.

Gross Errors

Gross errors are, in fact, not errors at all, but results of mistakes that are due to the carelessness of the observer. Pointing on the wrong survey target, incorrect reading on the scale, reading on the wrong scale, and wrong recording the value of reading, are some examples of blunders or mistakes. The blunders or mistakes generally result into large errors which can easily be detected compared to small errors.

Good field procedures are designed to assist in detecting the gross errors. These may be

(*a*) Careful checking of all pointings on survey targets.

(*b*) Taking multiple readings on scales and checking for reasonable consistency.

(*c*) Verifying recorded data by reading scales.

(*d*) Taking repeated measurements independently and checking for consistency.

(*e*) Using simple geometric and algebraic checks such as sum of all the angles taken at a station is 360°, sum of three angles in a plane triangle is 180°.

The gross errors must be detected and eliminated from the survey measurements before such measurements can be used.

Systematic Errors

The errors which are systematic or follow some pattern, are termed as *systematic errors*. Such errors are always according to some deterministic system and can be expressed by functional relationship. Measurement of distance by a too short tape and expansion of steel tape with respect to temperature, are the examples of systematic errors. Such errors are of a constant nature and hence can be computed using the functional relationships, and the measurement can be corrected for them.

The personal bias of an observer, and imperfect conditions of the instruments also lead to systematic errors. Systematic errors also occur through simplification of geometry such as considering a plane triangle instead of spherical triangle.

Like the gross errors, the systematic errors must also be detected and eliminated from the survey measurements before adjusting the remaining errors.

Random Errors

After all mistakes are detected and removed, and the measurements are corrected for all known systematic errors, there will still remain some variations in measurements that result from observational errors. These errors do not have any known functional relationship based on a deterministic system and have random behaviour, and therefore, are known as *random errors*. As stated earlier that a measurement is looked upon mathematically as a variable, the measurement having a component of error which behaves like a random variable, itself becomes a random variable.

Random errors, often called as accidental errors, are unpredictable in regard to both size and algebraic sign. They are truely accidental and cannot be avoided. The general behaviour of the random errors is that

(*i*) a plus error will occur as frequently as will a minus error,
(*ii*) small errors will occur more frequently than large errors, and
(*iii*) very large errors do not occur at all, or their chance to occur is remote.

Whereas systematic errors are dealt with mathematically using functional relationships or models, random errors use probability models.

Theory of errors discussed in Chapter 2 of *Higher Surveying*, deals with random errors.

2.5 CONCEPT OF PROBABILITY IN SURVEY MEASUREMENTS

A survey measurement finally left with only random errors, is a random variable just as number of dots on the top face of a dice. In order to understand the random nature of a survey measurement, it may be measured repeatedly and its frequency distribution describing its random variation is studied. A distance of about 510 m is measured 200 times, and the measurements free of gross errors and corrected for systematic errors, are presented in Table 2.1.

Table 2.1 Repeated measurements of a distance

Measured values of distance (*m*)	Number of measurements (frequency = *f*)	Relative frequency ($R = f/\Sigma f$)
510.11	1	0.005
510.12	3	0.015
510.13	7	0.035
510.14	18	0.090
510.15	21	0.105
510.16	35	0.175
510.17	39	0.195
510.18	30	0.150
510.19	23	0.115
510.20	10	0.050
510.21	11	0.055
510.22	0	0.000
510.23	2	0.010
	$\Sigma f = 200$	$\Sigma R = 1$

In Table 2.1, the relative frequency for a given interval is an indication of probability implied by the following equation.

$$P(E) \text{ [probability of a favourable outcome]}$$

$$= \frac{f \text{ [number of favourable outcomes]}}{n \text{ [number of all possible outcomes]}} \qquad \text{...(2.3)}$$

For example, for the inclusive interval 510.16 m to 510.169 m there are $f = 35$ observations. Therefore, the corresponding relative frequency is

$$\frac{f}{n} = \frac{35}{200} = 0.175$$

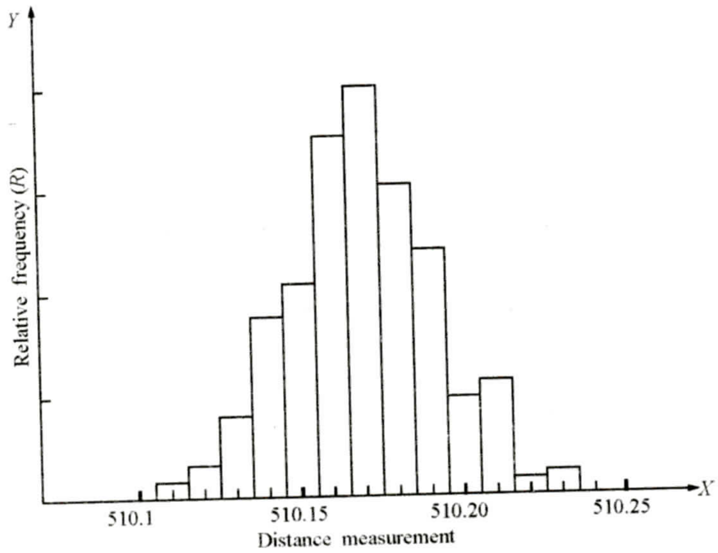


Fig. 2.2 Histogram

A diagram (Fig. 2.2), known as *frequency distribution* or more frequently *histogram*, can be constructed between the given class intervals and corresponding relative frequencies. It is found that the frequency distribution is centered on or near the value 510.175 m which is mean of all the readings. The highest frequencies of occurrence are at or near the central value. An interesting point to note is that the mean value has the highest relative frequency.

If the number of repeated measurements are increased to a very large number, and the class interval is made narrow, it will be found that each relative frequency approaches to a stable limit which is the *probability*. In such case the histogram approaches to the appearance of a curve, known as *probability distribution curve* or *probability density function curve* shown in Fig. 2.3. The area under the curve between two measured values of a distance, x_1 and x_2, is the probability of a measurement falling be

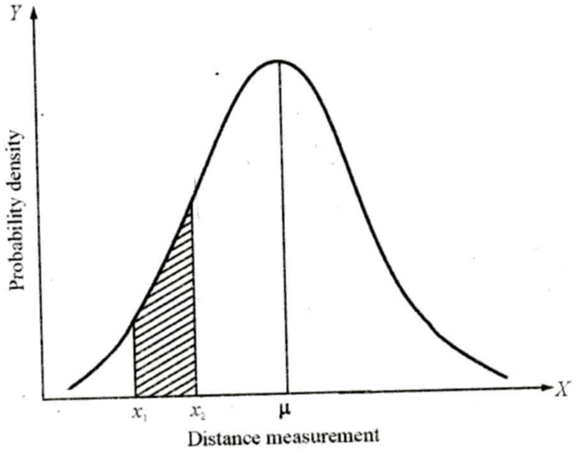


Fig. 2.3 Probability density function curve

tween the values x_1 and x_2. The central value μ represents the *distribution mean value* of the distance measurements. The distribution mean μ is taken as representative of the true value of the measured distance.

2.6 NORMAL DISTRIBUTION

The statistical analysis of survey measurements has indicated that the survey measurements follow *normal distribution* or *Gaussian distribution*. The normal distribution is expressed by

$$y = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\upsilon}{\sigma}\right)^2} \qquad \text{...(2.4)}$$

where y = the frequency of occurrence of residual,
υ = the residual,
σ = the standard deviation, and
e = the base of the natural logarithm ($e = 2.718$).

Fig. 2.4 shows a normal distribution curve.

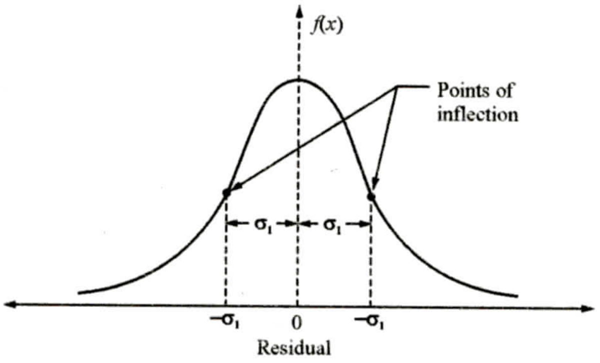


Fig. 2.4 Normal distribution curve

If the abscissa interval which is designated as the *class interval*, be denoted by I, and n be the number of measurements or residuals, then Eq. (2.4) takes the form

$$y = \frac{nI}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\upsilon}{\sigma}\right)^2} \qquad \text{...(2.5)}$$

The *standard deviation* σ, also known as *standard error*, is defined as

$$\sigma = \pm\sqrt{\frac{\Sigma\upsilon^2}{n-1}} \qquad \text{...(2.6)}$$

where n = number of observations.

The normal density function is characterized by two parameters, the distribution mean μ, and the standard deviation σ. For a large number of observations μ of residuals υ becomes zero. σ is also termed as *root-mean square* (R.M.S.) *error*. The square of standard deviation σ^2 is called the *variance*.

The standard deviation is a measure of the spread or dispersion of the probability distribution, and reflects the degree of variation in the measurement. If the repeated measurements are closely clustered together as shown in Fig. 2.5, they are said to have high precision and if they are widely spread apart, they have low precision.

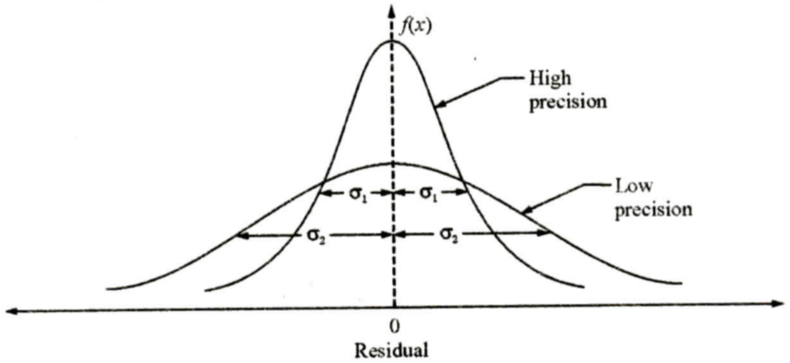


Fig. 2.5 Precision of measurements

There are some other indicators of precision, and are briefly given below.

Variance which is expressed as

$$V = \sigma^2$$

$$= \frac{\Sigma \upsilon^2}{n-1} \qquad \text{...(2.7)}$$

is also used as a measure of dispersion or spread.

Standard error of the mean expressed by

$$\sigma_m = \pm\sqrt{\frac{\Sigma \upsilon^2}{n\,(n-1)}}$$

$$= \pm\frac{\sigma}{\sqrt{n}} \qquad \text{...(2.8)}$$

is indicator of the limits of the errors found within which the true value of the mean will lie with a certainty of 68.3% of its being correct.

Standard error of the single measurement expressed by

$$\sigma_1 = \pm\sqrt{\frac{\Sigma \upsilon^2}{n-1}} \qquad \text{...(2.9)}$$

is equal to standard *error* σ. The two terms are used synonymously.

Most probable error expressed by

$$e = \pm\, 0.6745\sqrt{\frac{\Sigma \upsilon^2}{(n-1)}}$$

$$e = \pm\, 0.6745\,\sigma \qquad \text{...(2.10)}$$

is defined as the error for which there are equal chances of its being less than and more than the probable error. It is also called 50% error and expressed as E_{50}.

Most probable error of the mean is expressed as

$$e_m = \pm 0.6745\sqrt{\frac{\Sigma\upsilon^2}{n(n-1)}}$$

$$= \pm\ 0.6745\sigma_m \qquad \text{...(2.11)}$$

Maximum error corresponds to ± 3.29 σ. It is often used to separate mistakes or blunders from the random errors. If any error deviates from the mean by more than the maximum error, it is considered as a gross error, and that measurement is rejected. Different percentage errors are also sometimes used. They are expressed as

(*i*) E_{90} (90% error) $= \pm\ 1.645\sigma$ ...(2.12)
(*ii*) E_{95} (95% error) $= \pm\ 1.96\sigma$ ...(2.13)
(*iii*) $E_{95.5}$ (95.5% error) $= \pm\ 2.0\sigma$ ...(2.14)
(*iv*) $E_{99.7}$ (99.7% error) $= \pm\ 3.0\sigma$ ...(2.15)

ILLUSTRATIVE EXAMPLES

Example 2.1

The distance between two points *A* and *B* was measured 10 times under identical conditions, and the measured values were recorded as

53.56 m, 53.52 m, 53.54 m , 53.58 m, 53.55 m, 53.60 m, 53.54 m, 53.57 m, 53.54 m, 53.56 m.

Determine the following:

(*i*) The standard error
(*ii*) The standard error of the mean
(*iii*) The most probable error
(*iv*) The most probable error of the mean
(*v*) The variance
(*vi*) The maximum error.

Solution:

Determine the mean μ of the measurements, residuals υ, and υ^2 as shown in Table 2.2.

Number of observations $n = 10$

(*i*) Standard deviation

$$\sigma = \pm\sqrt{\frac{\Sigma\upsilon^2}{n-1}}$$

$$= \pm\sqrt{\frac{4.800\times 10^{-3}}{10-1}} = \pm\ \mathbf{0.023\ m}.$$

Table 2.2

Measurements (x) (m)	Residuals ($\upsilon = \mu - x$)	υ^2
53.56	– 0.004	1.600×10^{-5}
53.52	0.036	1.296×10^{-3}
53.54	0.016	2.560×10^{-4}
53.58	– 0.024	5.760×10^{-4}
53.55	0.006	3.600×10^{-5}
53.60	– 0.044	1.936×10^{-3}
53.54	0.016	2.560×10^{-4}
53.57	– 0.014	1.960×10^{-4}
53.54	0.016	3.600×10^{-5}
53.56	– 0.004	1.960×10^{-4}
$\mu = \dfrac{535.56}{10} = 53.556$	$\Sigma\upsilon = -0.000$	$\Sigma\upsilon^2 = 4.800 \times 10^{-3}$

(*ii*) Standard error of the mean

$$\sigma_m = \pm\sqrt{\frac{\Sigma\upsilon^2}{n(n-1)}}$$

$$= \pm\sqrt{\frac{4.800\times10^{-3}}{10\times(10-1)}} = \pm\ \mathbf{0.007\ m.}$$

(*iii*) Most probable error

$$e = \pm 0.6745\sigma$$

$$= \pm 0.6745 \times 0.023 = \mathbf{\pm 0.016\ m.}$$

(*iv*) Most probable error of the mean

$$e_m = \pm\ 0.6745\ \sigma_m$$

$$= \pm\ 0.6745 \times 0.007 = \pm\mathbf{0.005\ m.}$$

(*v*) Variance

$$V = \sigma^2$$

$$= (\pm\ 0.023)^2 = \mathbf{0.0005\ m^2.}$$

(*vi*) Maximum error

$$e_{\max} = \pm\ 3.29\sigma$$

$$= \pm\ 3.29 \times 0.023 = \pm\ \mathbf{0.076\ m.}$$

Example 2.2

The following eight measurements of a horizontal angle were made:

S.No.	Horizontal angle (A)
1	32°22′10″
2	32°21′50″
3	32°21′40″
4	32°22′00″
5	32°21′50″
6	32°22′00″
7	32°22′10″
8	32°22′20″

Compute the standard deviation, the most probable error, and the maximum error.

Solution:

The mean, $\mu = \frac{\Sigma A}{n} = 32°22'00''$

The residuals can be computed from

$$\upsilon = \mu - A$$

$$\upsilon_1 = +10'', \upsilon_2 = -10'', \upsilon_3 = -20'', \upsilon_4 = 0'', \upsilon_5 = -10'', \upsilon_6 = 0'', \upsilon_7 = +10'', \upsilon_8 = +20''$$

$$\Sigma\upsilon^2 = 1200$$

The standard deviation

$$\sigma = \sigma = \pm\sqrt{\frac{\Sigma\upsilon^2}{n-1}} = \pm\sqrt{\frac{1200}{7}}$$

$$= \pm\ \mathbf{13.1''}.$$

The most probable error of the mean

$$e = \pm\ 0.6745\sigma_m$$

$$= \pm 0.6745\frac{\sigma}{\sqrt{n}}$$

$$= \pm\ 0.6745 \times \frac{13.1}{\sqrt{8}}$$

$$= \pm\ \mathbf{3.1''}$$

The maximum error

$$e_{max} = \pm\ 0.29\sigma$$
$$= \pm\ 3.29 \times 13.1$$
$$= \pm\ \mathbf{43.1''}.$$

2.7 PROPAGATION OF ERROR

In surveying the two basic quantities which are measured, are angles and distances. Often these basic field measurements are used to calculate other quantities such as horizontal distances from measured slope distances, elevations from measured difference in height, areas, volumes, etc. The calculation of other quantities using the measured quantities is done through mathematical relationship between them. Since the measured quantities have errors, it is inevitable that the quantities computed from them will also have errors. Evaluation of the errors in the computed quantities as functions of the errors in the measurements, is called *error propagation.*

Let y be a quantity to be computed using a measured quantity x which has error dx through the relationship

$$y = ax + b \qquad ...(2.16)$$

The coefficient a and the constant b are assumed to be free of errors.

Now if x_t and y_t are the true values of x and y, respectively, then

$$x = x_t + dx \qquad ...(2.17)$$
$$y_t = ax_t + b \qquad ...(2.18)$$

So from Eq. (2.16) and Eq. (2.17)

$$y = a\ (x_t + dx)\ +\ b$$
$$= ax_t + adx\ +\ b$$
$$= (ax_t + b)\ +\ adx$$
$$= y_t + adx$$

If dy is the error in y, then from Fig. 2.6, we get

$$\tan\theta = \frac{dy}{dx} = a$$

or

$$dy = adx \qquad ...(2.19)$$

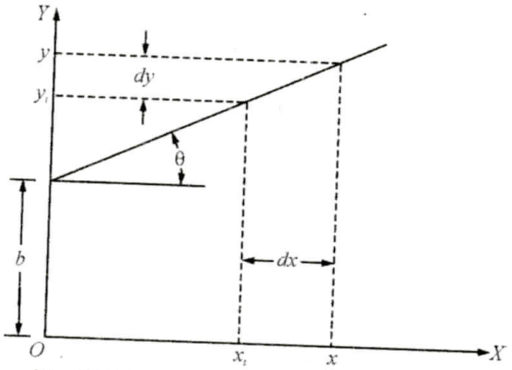


Fig. 2.6 Propagation of error in linear function

or $$dy = \left(\frac{dy}{dx}\right)dx \qquad \text{...(2.20)}$$

The Eq. (2.19) or (2.20) gives the error dy in the computed quantity y. In Eq. (2.19), the error function $dy = adx$ is linear in error dx in identical manner to $y = ax + b$. This will not be true in case of nonlinear functions such as $y = x^2$.

Let us take an example of a nonlinear function.

$$y = x^2 \qquad \text{...(2.21)}$$

$$y_t = x_t^2$$

or $$y = (y_t + dy) = (x_t + dx)^2$$

or $$y_t + dy = x_t^2 + 2x_t dx + (dx)^2$$

or $$dy = 2x_t dx + (dx)^2 \qquad \text{...(2.22)}$$

Neglecting $(dx)^2$ as dx is a small quantity, we get

$$dy = 2x_t dx \qquad \text{...(2.23)}$$

$$\frac{dy}{dx} = 2x_t$$

From Eq. (2.23), we find that $2x_t$ is the derivative of y with respect to x, evaluated at $x = x_t$.

$$\left(\frac{dy}{dx}\right)_{x=x_t} = 2x_t$$

Thus, from Eq. (2.22)

$$dy = \frac{dy}{dx}dx + (dx)^2 \qquad \text{...(2.24)}$$

Again neglecting $(dx)^2$, we get

$$dy = \frac{dy}{dx}dx \qquad \text{...(2.25)}$$

with the assumption that $(dx)^2$ is negligible.

Propagation of Standard Error

If the standard errors in the measured quantities are known, the standard error in the computed quantity can be evaluated. Let the standard errors in the measured quantities x_1, x_2, and x_3 be σ_{x_1}, σ_{x_2}, and σ_{x_3}, respectively. The quantity y is computed from x_1, x_2, and x_3.

Thus $y = f\,(x_1, x_2, x_3)$

The error in y is given by

$$dy = \frac{\partial f}{\partial x_1}dx_1 + \frac{\partial f}{\partial x_2}dx_2 + \frac{\partial f}{\partial x_3}dx_3 \qquad \text{...(2.26)}$$

and the standard error in y is obtained from the following equation.

$$\sigma_y^2 = \left[\frac{\partial f}{\partial x_1}\sigma_{x_1}\right]^2 + \left[\frac{\partial f}{\partial x_2}\sigma_{x_2}\right]^2 + \left[\frac{\partial f}{\partial x_3}\sigma_{x_3}\right]^2 \qquad \text{...(2.27)}$$

ILLUSTRATIVE EXAMPLES

Example 2.3

The dimensions of a trapezoidal land parcel is shown in Fig. 2.7. What is the ordinate y for a measured distance $x = 37.325$ m? If the error in the measured distance is 0.015 m, what would be the corresponding error in the calculated distance of y ?

Solution:

Let us represent

$$y = ax + b$$

From Fig. 2.7, we have

$$a = \tan\theta = \frac{EC}{DE} = \frac{80-50}{100} = 0.3 \text{ (exact)}$$

and $b = \text{AD} = 50$

As a and b have exact values, therefore, they are errorless

$$y_{(x\,=\,37.325)} = 0.3 \times 37.325 + 50$$

$$= \mathbf{61.1975\ m}.$$

and error in y for $dx = 0.015$ m, therefore,

$$dy = a.dx$$

$$= 0.3 \times 0.015$$

$$= \mathbf{0.0045\ m}.$$

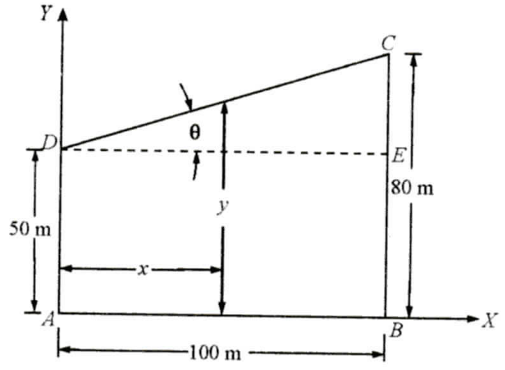


Fig. 2.7

1

Surveying—A Glimpse

INTRODUCTION

Surveying is defined as the art of determination of relative position of points on above or below the surface of earth by means of linear and angular measurement, distance, elevation and direction. The relative attitude of points is ascertained by levelling which is another branch of surveying. The maps of the different points of an area are drawn in plan or forms a vertical section with different scales which are essential for different engineering project like roads, highways, railroad, water supply, irrigation, transmission lines, bridges, buildings, dams and reservoirs. It is required for location of different geological structures, showing the geographical boundaries of district, states and countries.

Surveying further may be defined as the technique and science of accurately determining the terrestrial or three-dimensional space position of points and the distances and angles between them. These points are usually, but not exclusively, associated with positions on the surface of the earth, and are often used to establish land maps and boundaries for ownership or governmental purposes. In order to accomplish their objective, surveyors use elements of geometry, engineering, trigonometry, mathematics, physics, and law. An alternative definition, as per the American Congress on Surveying and Mapping (ACSM) is "the science and art of making all essential measurements to determine the relative position of points and/or physical and cultural details above, on, or beneath the surface of the earth, and to depict them in a usable form, or to establish the position of points and/or details".

Furthermore, as alluded above, a particular type of surveying known as land surveying (also per ACSM) is the detailed study or inspection by gathering information through observations, measurements in the field; questionnaires; or research of legal instruments, and data analysis in the support of planning, designing, and establishing of property boundaries. It involves the re-establishment of cadastral surveys and land boundaries based on documents of record and historical evidence, as well as certifying surveys (as required by statute or local ordinance) of subdivision plats/maps, registered land surveys, judicial surveys, and space delineation. Land surveying can include associated services such as mapping and related data accumulation, construction layout surveys, precision measurements of length, angle, elevation, area, and volume, as well as horizontal and vertical control surveys. The analysis and utilization of land survey data and its requirement in the planning and execution of nearly every form of construction are very much essential. The most familiar modern uses of surveying are in the fields of transport, building and construction, communications, mapping, and defining legal boundaries for land ownership.

The techniques of conventional surveying and mapping have been revolutionised with the advent of variety of sophisticated instruments in the modern system of surveying and mapping. Such modern instruments and terminology associated with modern surveying are Electronic Distance Measurements (EDM) instruments, Laser (light amplification by stimulated emission of radiation), theodolite, electronic field book, Inertial Positioning System (IPS), Global Positioning System (GPS), Digital Terrain Models (DTMs), automated photographic system, Geographic Information System (GIS), Doppler positioning system and so on.

1.1 HISTORICAL BACKGROUND OF SURVEYING

Surveying has been an essential element in the development of the environment since 5000 years ago. In ancient Egypt, when the Nile river overflowed its banks and washed out farm boundaries, they were reconstructed through the application of simple geometry. Perfect squareness and north-south orientation of the Great Pyramid of Giza built in 2700 BC, affirmed the Egyptians' command of surveying. The Egyptian land register goes back to 3000 BC. Under the Romans, land surveyors were established as a profession, and they established the basic measurements under which the empire was divided, such as a tax register of conquered lands in 300 AD. In England, The Domes Day by William, the Conqueror, in 1086, covered all England, contained names of the land owners, area, land quality, and specific information of the area's content and habitants. Continental Europe's Cadastre was created in 1808 founded by Napoleon Bonaparte. It contained numbers of the parcels of land, land usage, names, etc., and value of the land. Some 100 million parcels of land, triangle survey, measurable survey, map scale: 1:2500 and 1:1250 spread fast around Europe, but faced problems especially in Mediterranean countries, Balkan, and eastern Europe due to cadastre upkeep costs and troubles. A cadastre loses its value if register and maps are not constantly updated. Large-scale surveys are a necessary pre-requisite to map-making. In the late 1780s, a team from the Ordinance Surveys of Great Britain, originally under General William Roy began the principal triangulation of Britain using the specially built Ramsden theodolite.

The Romans developed improved methods of surveying during Roman period. They developed measuring wheel called **odometers** to measure distances. The main development of surveying took place in 19th century after the invention of telescope. More excellent instruments like magnetic compass, levelling instruments and theodolites were developed for making measurement quite quickly and efficiently.

Further development of methods and surveying instruments took place in the 20th century. Precise levelling instruments and modern optical theodolites were invented. Electronic Distance Measurement (EDM) instruments were introduced for accurate measurement of distances. The use of computer, new method of photogrammetric survey, the use of lasers, satellites, applications of GIS, GPS, DTM, remote sensing has brought the subject of surveying to a new and very sophisticated level of study.

1.2 DIVISIONS OF SURVEYING

The shape of earth is an approximate oblate ellipsoid of its revolution. The lengths polar and equatorial axes are 12,713.168 km and 12756.413 km, respectively. The difference is only about 43 km as computed by Clark in 1866. The polar axis is shorter than the equator axis. When the distance of measurement is small, practically there is no difference between the curved distances and corresponding straight line distances, and curvature of the earth is neglected. But in case of bigger area long distances cannot be neglected and earth curvature is considered. Hence, the surface of the earth has the curvature and this is the main factor on which the following two disivisions of surveying is based:

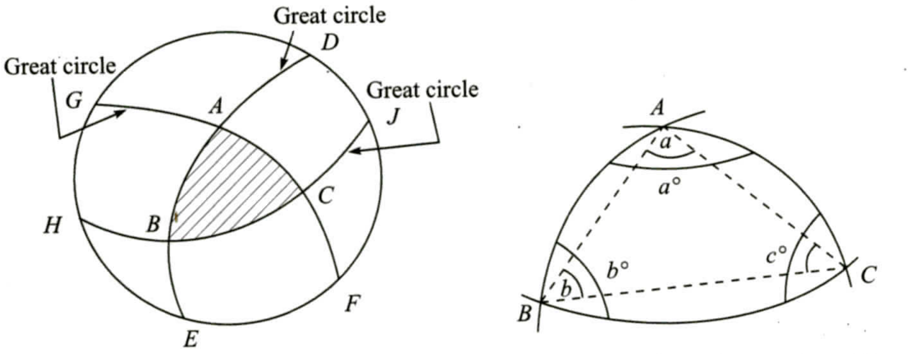
1.2.1 Plane Surveying

In plane surveying the curvature of the earth is not taken into account and the surface is treated to be plane. Normally, in small surface area, error due to curvature is significantly much smaller. It can be applied quite safely up to an area of 250 km^2. Plane surveying is even allowed to conduct to an area upto about 1000 km^2. Most of the day to day surveys are based on plane surveying. It may be mentioned that the difference between an arc of 18.5 km on the surface of the earth and corresponding chord distance is only 10 mm. Similarly, sum of the angles of spherical triangle having an area of 200 km^2 on the earth surface and that of the corresponding angles of plane triangle is only 1 second, and therefore, for small areas, surveying conducted is mainly plane surveying.

1.2.2 Geodetic Surveying

Geodetic surveying is that type of surveying where the curvature of earth is considered. It is essential if the area of survey is quite large. Actual curvature of the earth is taken into account in this type of survey. The lines on the earth surface are curved; and triangles are spherical. It, therefore, invites spherical trigonometry. Science of geodesy is required in this type of survey. The main object of geodetic surveying is to determine the precise location of widely spaced on the surface of the earth.

Figure 1.1(a) shows three points *A*, *B* and *C* on the mean surface of the earth. The lines *DABE*, *HBCJ* and *GACF* are the arcs of the great circles passing through the centre of the earth. The intersection of a plane passing through the centre of the earth forms the great circle. In Figure 1.1(a), the triangle *ABC* shown by shaded line is the spherical triangle. In Figure 1.1(b) the enlarged view of the spherical triangle *ABC* of Figure 1.1(a) is depicted. A plane triangle *ABC* is formed by the dotted lines. The angles *a*, *b* and *c* are the angles of the plane triangle and angles $a°$, $b°$ and $c°$ are the corresponding angles of the spherical triangle. These angles $a°$, $b°$ and $c°$ are greater than the angles *a*, *b* and *c* of the plane triangles. The distances *AB*, *BC* and *CA* of spherical triangles in geodetic survey are determined by using spherical trigonometry and in plane triangle by using plane trigonometry.



(a) Spherical triangle *ABC* on earth surface shown by shaded lines.

(b) Sperical triangle in an enlarged form.

Figure 1.1 The spherical triangle.

Thus, above discussion of these two types of surveying may be concluded as:

(a) Plane surveying:

(i) It is used for relatively small arcs.
(ii) A curved line on the surface of earth is taken as a straight line.
(iii) The directions of plumb lines at various points are assumed to be parallel.
(iv) The spherical angles are considered as plane angles.
(v) The standard of accuracy is lower than that of geodetic surveying.

(b) Geodetic surveying:

(i) It is used for large areas.
(ii) It is used for establishing precise control points.
(iii) The surface of the earth is considered in its exact shape with curvature.
(iv) The direction of plumb lines at various points are different.
(v) The accuracy is very high. The instruments used are precise.

1.3 CLASSIFICATION OF SURVEYING

The classification of surveying may be based on four dimensions, they are: the function and purpose of surveying; instruments used; method employed; and the place of work.

1.3.1 Classification based on Function and Purpose of Surveying

(i) Land survey is for determination of areas or boundaries. It may be also called property survey boundary survey or cadastral survey.
(ii) City survey is for urban planning. It is required for layout of roads, street, sewers, water supply, and layout of buildings in urban areas.
(iii) Topographic survey is for determination of region such as hills, rivers, lakes, streams, forest, man made features like roads, villages, buildings, canals, and so on.

(iv) Route surveys are those types which include planning, design, execution of highways, railways, pipe lines canal etc.

(v) Engineering survey is adopted for engineering purposes. It is done for collection of requisite data for planning design and execution of engineering project like dams, reservoirs tunnels, spillways, roads, bridges, power house, transmission lines, etc. Engineering survey has further stages like reconnaissance preliminary and final location surveys.

(vi) Military survey is conducted for military purposes for determining points of strategic importance.

(vii) Mine survey is used for exploring mineral wealth deposited in earth's structure and to guide tunnelling and other operations associated with it.

(viii) Satellite survey is conducted to establish inter continental inter datum and inter islands geodetic tics all over the world by making observations on artificial satellites.

(ix) Geological survey is used to determine and explore the components of earth's crust. Geological survey is further conducted for the purpose of recording the geologically significant features of the area under investigation. In the past, in remote areas, there was often no base topographic map available, so the geologist also needed to be a competent surveyor to produce a map of the terrain, on which the geological information could then be draped..More recently, satellite imagery or aerial photography is used as a base, where no published map exists. Such a survey may also be highly specialised - for instance focusing primarily on hydrogeological, geochemical or geomagnetic themes. (Do not confuse this term with Geological survey, typically a government (national, regional or local) body, charged with maintaining and improving the record of the geology of the area in which it operates

(x) Astronomical surveys are made for determination of attitude, longitude, azimuths, local time, etc., of heavenly bodies such as sun and other fixed star.

(xi) Control survey is to establish horizontal and vertical positions of widely spaced control points using the principles of geodetic survey.

(xii) Archaeological survey is done to unearth the relics of antiquity. This survey is used also to accurately assess the relationship of archaeological sites in a landscape or to accurately record finds on an archaeological site.

(xiii) Gravity survey is carried out to measure the force of gravity at various points.

(xiv) Foundation survey done to collect the positional data on a foundation that has been poured and is cured.

(xv) Mortgage survey or physical survey is a simple survey that generally determines land boundaries and building locations. Mortgage surveys are required by title companies and lending institutions when they provide financing to show that there are no structures encroaching on the property and that the position of structures is generally within zoning and building code requirements. Some jurisdictions allow mortgage surveys to be done to a lesser standard, however, most US state minimum standards require the same standard of care for mortgage surveys as any other survey. The resulting higher price for mortgage surveys has led some lending institutions to accept mortgage inspections not signed or sealed by a surveyor.

(xvi) Bathymetric survey is a survey carried out to map the seabed profile.

(xvii) Deformation survey is a survey to determine if a structure or object is changing shape or moving. The three-dimensional positions of specific points on an object are determined; a period of time is allowed to pass. These positions are then re-measured and calculated, and a comparison between the two sets of positions is made.

(xviii) Soil survey or soil mapping is the process of determining the soil types or other properties of the soil cover over a landscape, and mapping them for others to understand and use.

(xix) Wetlands delineation and location survey is a survey that is complete when construction work is to be done on or near a site containing defined wetlands. Depending on the local, state, or federal regulations wetlands are usually classified as areas that are completely inundated with water more than two weeks during the growing season

1.3.2 Classification based on Instruments Used

chain-Cloth

(i) Chain survey is the simplest type in which linear measurement are made by the chain and the tape. Angular measurements are not required here.

(ii) Compass survey is a survey in which angles are to be measured by a magnetic compass and linear measurement by a chain or a tape.

(iii) Plane table survey is a survey in which a map of a plot of land is to be prepared in the field in a drawing tapes on the plane table.

(iv) Levelling survey is used to determine the elevations of different points of a plot with the help of an instrument called level.

(v) Chain and compass survey is used to plot of a land with the help of chain and compass.

(vi) Theodolite survey is primarily used in traversing and triangulation for providing controls. The instrument theodolite is used to measure vertical and horizontal angles for plotting curves on roads railways gate.

(vii) Tacheometric survey is done with special type of instrument known as tacheometer to determine horizontal and vertical distances.

(viii) Photogrammetry or photographic survey is made with the help of photographs.

(ix) EDM survey is done by Electronic Distance Measurement (EDM) instrument method. It has been developed quite recently. Instruments are for high waves and radio wave measurement.

1.3.3 Method Employed

(i) Triangulation survey is the process of measuring the angles of a chain or network of triangles formed by stations marked on the surface of earth.

(ii) Traverse survey is that type of survey in which number of survey lines forms the framework and the directions and the lengths of survey lines are measured with the help of angle or direction measuring instruments.

1.3.4 Place of Work

(i) Land survey is done to measure objects on earth's surface.

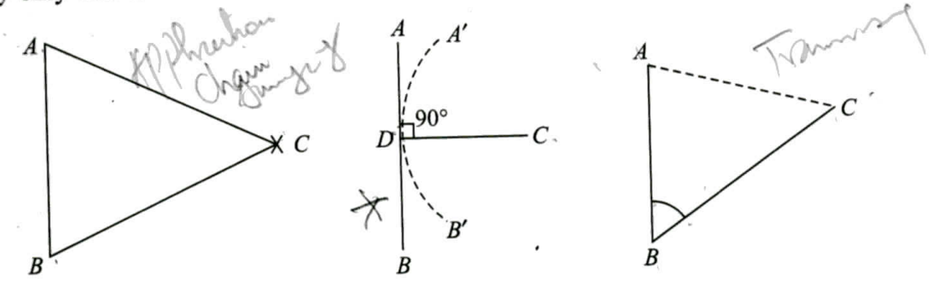
(ii) Hydrographical or marine survey is done to measure objects under water.
(iii) Aerial survey is done via aeroplane in air.

1.4 PRINCIPLES OF SURVEYING

The basic principles of surveying have changed over ages, but the tools used by surveyors have evolved tremendously. Engineering, especially Civil Engineering depends too much on surveyors. The surveyors should have a thorough knowledge of algebra, basic calculus, geometry and trigonometry. In addition to the above, surveyors should be able to use all delicate types of instruments with precision and accuracy. The methods of plane surveying are based on the two fundamental principles, as follows:

1.4.1 Locating a Point from Two Other Known Points

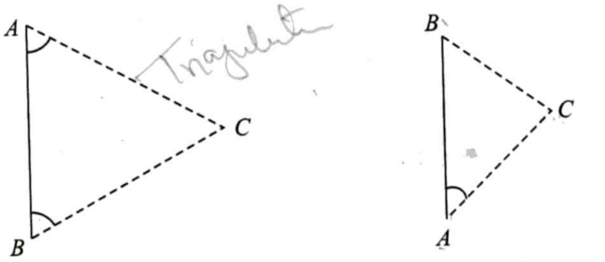
The relative positions of points in survey works can easily be located from two known fixed points by only one of the methods described in Figure 1.2(a), (b), (c), (d) and (e).



(a) Location of *C* by swinging method.

(b) Location of *D* from *C* by swinging.

(c) Locating point *C* by measuring angle *ABC*.



(d) Location of *C* by measuring angles *BAC* and *ABC*.

(e) Location of *C* by measuring angles *BAC* and distance *DC*.

Figure 1.2 Methods of locating a point from two other known points.

(i) In Figure 1.2(a) the location of points *A* and *B* on the ground are known. The locations of point *C* can be located easily by swinging arc method by measuring *AC* and *BC* is commonly used in chain survey.

(ii) In Figure 1.2(b) a perpendicular *CD* at point *D* on *AB* from *C* can be drawn by a tape taking minimum distance of CD to AB by swinging the tape. The tape is kept on swinging along the path *A′DB′* shown by a dotted line and point *D*, i.e. *CD* is the minimum distance of *C* to the chain line. Thus, *CD* is perpendicular to *AB* at *D* from *C*. This can be plotted by set squares.

(iii) In Figure 1.2(c) the distance *AC* and angle *CBA* can be measured. Using a protractor or trigonometrical equation point *C* is plotted. It is used in traverse survey.

(iv) In Figure 1.2(d) the angle *BAC* and *ABC* are measured by an angle measuring instrument. Knowing the distance *AB*, *C* is located either by protractor or by solution of trigonometrical equation of triangle. It is used in triangulation survey. Thus, the points can be plotted in drawing sheet to a suitable scale

(v) In Figure 1.2(e), angle *BAC* and distance *BC* are measured and point *C* is plotted by protractor or trigonometrically or by swinging about the point *C*. This method is of minor utility and is used in exceptional cases.

1.4.2 Working from the Whole to the Port

In this procedure, surveyor should first establish a sufficient number of points with high degree of precision in and around the area to be surveyed. These are called **primary control point.** The gaps are then filled with secondary control points at closer interval. Further, gaps are then filled up by tertiary control points even at closer intervals. For the survey of ordinary nature, the tertiary control points are used to fix details on the ground. The aim of working in this way is to prevent accumulation of error.

1.5 BASIC MEASUREMENTS IN SURVEYING

The reference line in surveying is the direction of gravity which is required in all measurements. This direction of gravity line is the plumb line, i.e. the line of freely suspended plumb bob which is taken as a vertical line. The plane perpendicular to this gravity line is called **horizontal line.** Surveying basically consists of the following measurements:

1. Horizontal distances: The distance along a horizontal plane is the horizontal distance. If a distance is measured in a slope, it is converted to its horizontal equivalent.
2. Vertical distances: Vertical distance is measured along the direction of gravity. It is essential to find the difference of levels between points.
3. Horizontal angle: Horizontal angle is the angle between two lines in a horizontal plane.
4. Vertical angle: It is the angle between two lines in the same vertical plane.
5. Vertical plane: A plane containing a vertical line is a vertical plane.
6. Level surface: It is the equipotential surface of the earth's gravitational field. It is actually a curve surface and every element of which is normal to the plumb line. A body of still water is the best example of level surface.
7. Level line: A line lying on a level surface is a level line. Thus, it is a curved line normal to the plumb at all points.
8. Horizontal plane: It is a plane which is tangential to the level surface at a particular point.

9. Horizontal line: A line tangent to the level surface is a horizontal line.
10. Vertical line: It is a line perpendicular to the horizontal plane.
11. Zenith angle: An angle between two lines in a vertical where one of the lines directed towards the zenith is known as **zenith angle**.
12. Elevation: The vertical distance of a point from an assumed datum or mean sea level (MSL) is known as an **elevation**.
13. Contour: It is an imaginary line of equal or constant elevation on the surface of the ground.
14. Grade or gradient: The rate of ascent or descent or the slope of the line is called **grade or gradient**.
15. Latitude: If the y-axis in a Cartesian co-ordinate is in north-south direction, y co-ordinate of a point is called **latitude**.
16. Departure: If the x-axis in a Cartesian co-ordinates is in east-west direction, x co-ordinate of a point is called **departure**.

1.6 INSTRUMENTS USED FOR DIFFERENT MEASUREMENTS

Different instruments are used for different measurements as follows:

(i) Instruments used for horizontal distances are tapes, chains, tacheometers and EDM.
(ii) Instruments used for vertical distances are levelling instruments, tacheometers, and so on.
(iii) Instruments used for horizontal angles include magnetic compass, theodolites, sextants, etc.
(iv) Instruments used for vertical angles are theodolites, clinometers and sextants.

1.7 BASIC METHODS USED IN SURVEYING

In surveying, all the methods involve in measuring angles and distances are for both horizontal and vertical planes. The locations of points required to measure are determined by the applications of various trigonometrical and geometrical processes. In surveying, these processes are known by distinctive names as follows:

1.7.1 Methods for Fixing Position in the Horizontal Plane

Three methods are used for fixing the position. They are intersection, angle and distance and resection.

Intersection

Intersection can be done by intersection of angles (Figure 1.3) and by intersection by distances (Figure 1.4). In the intersection of angles, the points *A* and *B* can be located and its distance can be measured. The position of point *C* can be obtained by intersection relative to *A* and *B*. The angle *CAB* and *CBA* can be measured in the field and position of *C* is obtained by plotting the

6

LEVELLING

6.1 GENERAL

For the execution of engineering projects, such as highways, railways, canals, tunnels, dams, irrigation and drainage works, the elevations (*cf.,* Sec.1.5) of different points along the alignment of the project are required. *Levelling* is the operation in surveying performed to determine and establish elevations of points on the surface of the earth or beneath the surface of the earth, to determine differences in elevation between points, and to control grades in construction surveys. Success of the engineering projects such as above, depends upon accurate determination of elevations. Levelling is of prime importance to the engineers, both for design as well as for execution of a project.

Levelling is a branch of surveying which deals with measurements in vertical planes.

6.2 DEFINITIONS OF SOME BASIC TERMS

Some of the terms, *e.g.,* level surface, horizontal plane, elevation, vertical angle, etc., have already been defined in Sec.1.5. Definitions of some of the terms used in levelling are given below.

Datum or datum surface: It is a level surface with respect to which the levels of points are measured or referred to. It may be a standard surface like mean sea-level or an arbitrary surface.

Bench Mark (B.M.): A bench mark is a permanent or semi-permanent physical mark of known elevation. It is set as point of reference in order to provide a point of beginning for determining elevations of other points. A bench mark is a bronze plate either in the top of a concrete pillar or in the foundation of a structure. Other locations for bench mark may be the top of a culvert headwall, the top of an anchor bolt, or the top of a spike driven into the base of a tree. The bench marks are established by government agencies for the convenience of the surveyors and civil engineers. The surveyors or civil engineers may also establish temporary bench marks for their own use.

6.3 CURVATURE AND REFRACTION

Since levelling is measured in vertical plane, the effects of curvature of the earth and atmospheric refraction which occur in vertical direction, must be taken into account. The required correction to the observations are either determined by the appropriate calculations or else by the measurement techniques designed to eleminate these effects.

It is evident from the definitions that a horizontal line departs from a level surface because of the curvature of the earth as shown in Fig. 6.1.

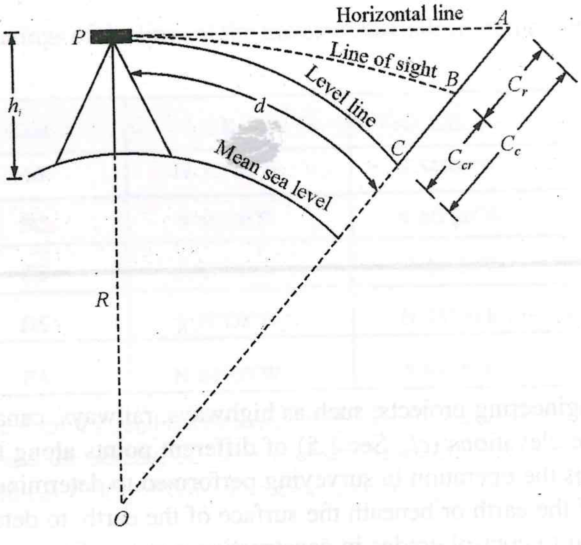


Fig. 6.1 Curvature and refraction

Since the line of sight cannot follow a curved line, an error is introduced and objects appear lower than they really are. This causes increase in reading and, therefore, this correction is negative.

Similarly, the effect of atmospheric refraction is to bend light rays slightly downward as they pass through layers of air of different densities. Light rays reflected from objects and incident on the retina enable us to see them. But the eye perceives them only in the direction tangent to the line of sight at the point of incidence, so the effect of refraction is to make objects appear higher than they really are. Since due to the effect of refraction reading is decreased, the correction for refraction is positive.

In Fig. 6.1, *AB* is the correction for refraction (C_r) and *AC* is the correction for earth curvature (C_c). If the distance from the point of tangency *P* to the point in question in kilometres is *d* and the mean earth's radius is *R* then from the right-angled triangle *OPA*, we have

$$(R + h_i)^2 + d^2 = (R + h_i + C_c)^2$$

where h_i is the height of the instrument axis above the mean sea level. Whilst the value of *R* is approximately 6370 km, the value of h_i will be of the order of 1.4 m, and therefore, can be neglected compared to the value of *R*.

Thus

$$(R + h_i)^2 + d^2 = (R + h_i)^2 + C_c^2 + 2(R + h_i)\mathrm{C_c}$$

$$d^2 = C_c^2 + 2(RC_c + h_iC_c)$$

Since C_c is very small, the term C_c^2 and h_iC_c will be very small and can be neglected. Therefore,

$$d^2 = 2RC_c$$

or $C_c = -\dfrac{d^2}{2R}$ (taking sign of the correction) ...(6.1)

Under the normal atmospheric conditions, C_r is one seventh of C_c but of opposite sign, *i.e.*,

$$C_r = +\frac{1}{7}C_c \qquad \text{...(6.2)}$$

Since the effects of curvature and refraction are in opposite directions to each other, the combined correction $BC = AC - AB$ will be

$$C_{cr} = C_c - \frac{1}{7}C_c$$

$$= \frac{6}{7}C_c = \frac{6}{7}\left(-\frac{d^2}{2R}\right) = -\frac{3}{7}\frac{d^2}{R} \qquad \text{...(6.3)}$$

Taking R as 6370 km, the combined correction is

$$C_{cr} = -\ 0.0673d^2 \qquad \text{...(6.4)}$$

In Eq. (6.4), d is expressed in kilometre and C_{cr} in metre.

In ordinary levelling work, the line of sight is seldom more than 100 m long and the corresponding value of C_{cr} is only 0.7 mm. Thus curvature and refraction are rarely significant except in precise levelling.

Distance to the Visible Horizon

In Fig. 6.2, let P be the point of observation at height h and A be the point on the *horizon*, *i.e.*, a point where the tangent from P meets the level line. Taking value of R as 6370 km and the mean coefficient of refraction as 0.071 for sights over land, the distance to visible horizon is given by

$$d = 0.0483\sqrt{Rh} \text{ kilometres} \qquad \text{...(6.5)}$$

$$\text{or} \quad d = \sqrt{\frac{h}{0.0673}} \text{ kilometres} \qquad \text{...(6.6)}$$

where h is in metres.

In Eqs. (6.5) and (6.6), both curvature and refraction corrections are taken into account.

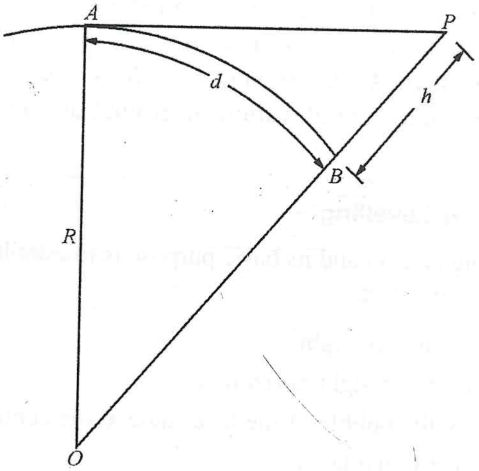


Fig. 6.2 Visible horizon

6.4 METHODS OF LEVELLING

Difference in elevations may be determined by the following methods:

1. Direct differential or spirit levelling
2. Indirect or trignometric levelling
3. Stadia levelling
4. Barometric levelling.

Direct Differential or Spirit Levelling

Differential levelling or spirit levelling is the most accurate and simple direct method for determining the difference of elevations between two points using an instrument known as *level* with a *levelling staff*. In Fig. 6.3, A is the point of known elevation, say h_A, with respect to which the elevation h_B of the point B is to be determined. The level is set up between the two points and levelled with the help of a level tube attached to the level. When the instrument is levelled, the line of sight would be horizontal. The reading S_A is taken on the levelling staff by first putting it on A and then S_B by putting on B.

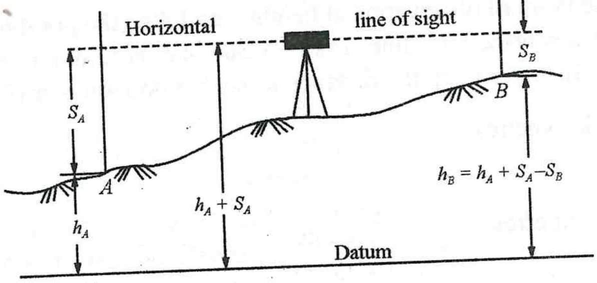


Fig. 6.3 Direct differential levelling

The elevation of the line of sight will be $h_A + S_A$ and the elevation of B will be $h_A + S_A - S_B$. Hence, the difference in elevation $\Delta h_{BA} = h_B - h_A = (h_A + S_A - S_B) - h_A = S_A - S_B$. If $(S_A - S_B)$ is positive, the point B is higher than A and if negative, B is lower than A. It shows that to determine the difference in elevations, the elevation of ground at which the level is set up need not be considered.

Instrument Used in Differential Levelling

The instrument used in levelling is level and its basic purpose is to establish a horizontal line of sight. It consists essentially of the following parts:

1. A telescope which provides a line of sight.
2. A level tube for making the line of sight horizontal.
3. A levelling head for bringing the bubble of the level tube at the centre of its run.
4. A tripod for providing support to the level.

The another instrument which is used with the level to determine amount by which a point is above or below the line of sight, is a levelling staff, straight rectangular rod having graduations and the foot of the staff representing zero reading. The vertical distance between the line of sight and the point over which the staff is held vertically, is the *staff reading*.

Types of levels

Of the various types of levels the following are the most common:

1. Dumpy level
2. Tilting level.

The modern levelling instruments used for precise works or geodetic surveys are:

1. Self-levelling or Automatic level
2. Geodetic level.

The instruments used in reconnaissance surveys to have an idea of change in levels are:

1. Hand level
2. Clinometer.

A brief description of the above levels is given below.

Dumpy level

The dumpy level shown in Fig. 6.4, consists of a telescope, generally the internal focussing type, fixed on a vertical spindle. The telescope tube and the vertical spindle are cast as one piece. The spindle revolves in the socket of the levelling head. The levelling head consists of two parallel plates held apart by three (or four) levelling screws. The upper plate is called tribrach and the lower one is called trivet stage. The telescope can be rotated in the horizontal plane about its vertical axis.

A sensitive level tube is fitted on the top of the telescope or on its side. An inclined mirror (not shown in the figure) is attached to the level tube to enable the observer to view the bubble from the eye end of the telescope without moving round the instrument.

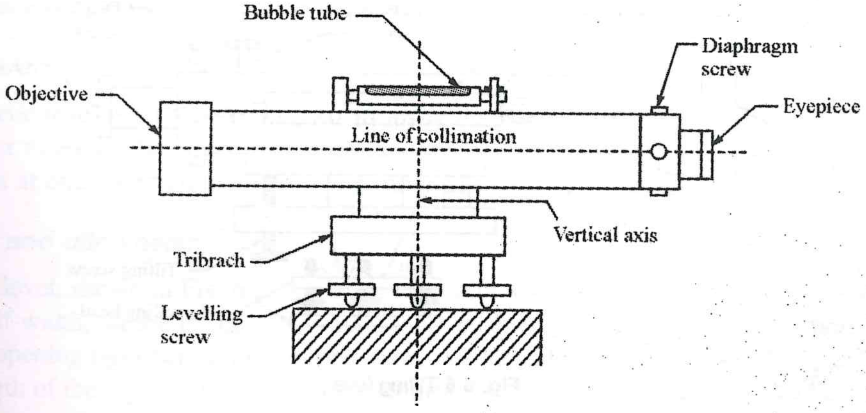


Fig. 6.4 Dumpy level

The cross hairs of the diaphragm normally have a vertical line and a horizontal line. When the instrument is in adjustment, the line of sight of the instrument is perpendicular to the vertical axis and parallel to the bubble tube axis as shown in Fig. 6.5.

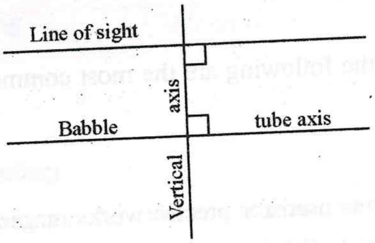


Fig. 6.5 Relationship of the line of sight with the vertical axis and bubble tube axis

The modern dumpy levels are provided with a clamp for clamping the telescope to the tribrach in any desired position. Also a graduated horizontal circle is provided to determine the direction of the telescope.

The dumpy level is simple in construction and requires fewer permanent adjustments. Adjustments once carried out remain for a long period. Before taking reading, the instrument requires temporary adjustments. These are (a) setting up, (b) levelling up, and (c) elimination of parallax, discussed in Sec. 4.4.4. A dumpy level is most useful when from one setting of the instrument, levels of several points are to be determined.

Tilting level

The telescope of a tilting level is not rigidly fixed to the vertical spindle as in the case of dumpy level. The telescope can be tilted on a pivot about a horizontal axis in the vertical plane upwards or downward through a small angle by means of a tilting screw as shown in Fig. 6.6.

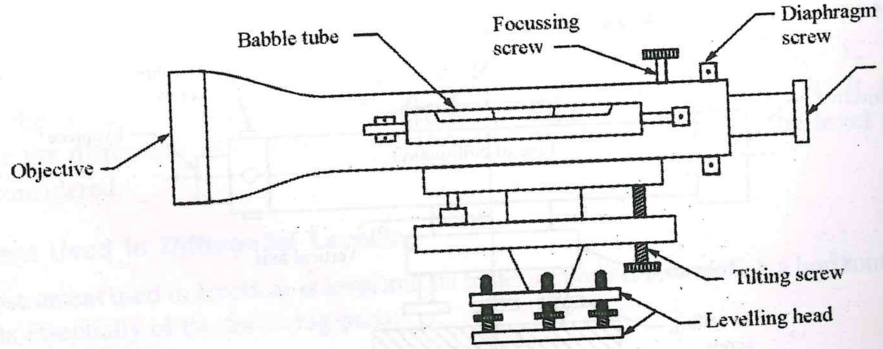


Fig. 6.6 Tilting level

The bull's eye or circular level is fixed to the upper plate of levelling head for approximate levelling by foot screws. The exact levelling of the instrument is done using the tilting screw before taking every reading. The tilting screw is usually graduated to set out gradient lines.

Tilting levels are more robust, compact and accurate than dumpy levels. These have shorter telescope and are lighter. The tilting arrangement saves time required for temporary adjustments. A tilting level is most useful when only a few readings are to be taken from one setting of the instrument.

Automatic level

The automatic level employs a gravity referenced prism or mirror compensator to automatically orient the line of sight (line of collimation). The instrument is quickly levelled when a circular spirit level is used. When the bubble has been centered or nearly so, the compensator takes over and maintains the horizontal line of sight, even if the telescope is slightly tilted. Fig. 6.7 shows the section through an automatic level.

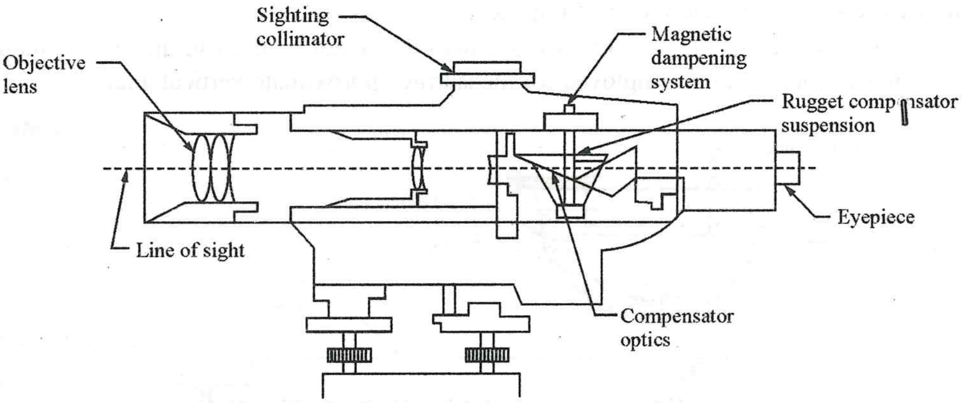


Fig. 6.7 Section through an automatic level

Automatic levels are extremely popular in present-day surveying operations. They are quick to set up, easy to use, and can be obtained for use at almost any required precision.

Geodetic level

The geodetic level is a tilting level used for extreme precision required in first-order levelling. It is made of invar to lessen the effect of temperature and equipped with stadia hairs for determining three staff readings at one observation and for determining lengths of the sights.

Hand level and clinometer

The hand level, shown in Fig. 6.8, is a brass tube with a small level tube mounted on the top. A 45° mirror of half width, on the inside of the main tube, enables the user to view the bubble by reflection through the opening right beneath the bubble tube. As the staff viewed through the level is not magnified, the length of the sight is limited by the visibility of staff readings with the naked eye.

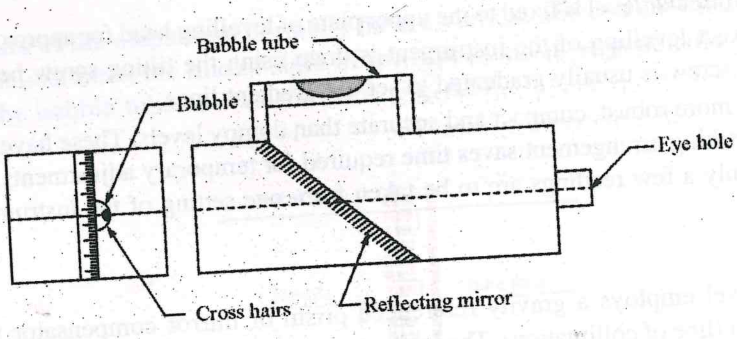

Bubble tube
Bubble
Eye hole
Cross hairs
Reflecting mirror

Fig. 6.8 Hand level

The hand level is used in reconnaissance surveys when the extreme accuracy is not required, and in taping to hold the tape horizontally. It is also used for estimating how high or low the dumpy level must be set in order to be able to read the levelling staff.

The clinometer shown in Fig. 6.9, known as Abney level can be used in the similar manner as the hand level. In addition, it can be employed for measuring approximate vertical angles.

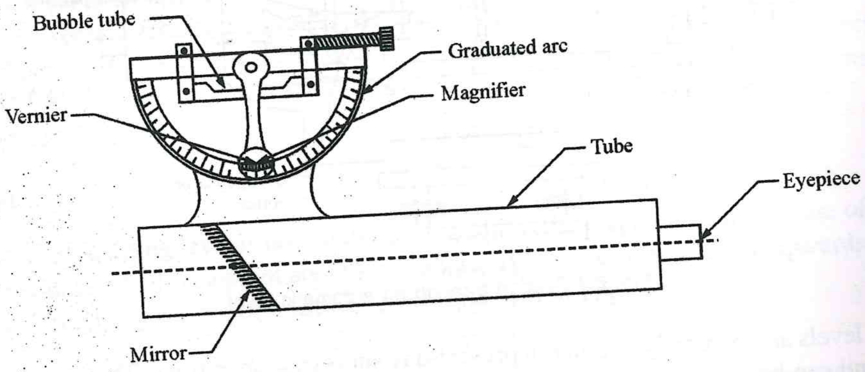

Bubble tube
Graduated arc
Magnifier
Vernier
Tube
Eyepiece
Mirror

Fig. 6.9 Clinometer

Types of levelling staffs

The levelling staffs are classified as:

1. Self-reading staff, and
2. Target staff.

A self-reading staff is the one which can be read directly by the instrument man through the telescope. On the other hand, a target staff contains a moving target against which the reading is taken by staff man.

Self-reading staff

There are usually three types of self-reading staffs.

Solid staff or single-piece staff

The single-piece or solid staff shown in Fig. 6.10, is a single long flat rod of well-seasoned wood, about 75 mm wide and 25 mm thick, with a metal shoe at the bottom, and graduated on one face. The smallest division is of 5 mm in metric system. The length of the staff is generally 3 m.

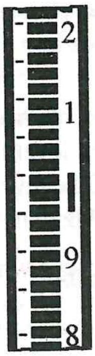


Fig. 6.10 Self-reading or solid staff

Folding staff

A folding staff is hinged in the middle so that top half can be folded over to lie against the lower half. When fully extended, the upper part is held firmly in position by a metal pin passing through holes in plates at the side of the hinge. When the staff is not in use it can be folded about the hinge so that it becomes convenient to carry it from one place to the other.

Telescopic staff (sopwith pattern)

A telescopic staff consists of two hallow pieces and one solid top piece, each telescoping into the bottom one. The telescopic staff has advantage over the other types that greater lengths of staff are obtainable up to 4 m to 5 m. When the staff is folded up, it is only a little longer than the lowest piece. When the upper sections are pulled out for use, they are engaged and held by brass spring-catches.

Target staff

A target staff has a sliding target equipped with vernier which can be clamped at any point along the staff. If it is made in more than one piece, separate verniers are provided for each piece.

For taking the reading, the level man directs the staff man to raise or lower the target till it is bisected by the line of sight. The staff man then clamps the target and takes the reading.

Relative merits of self-reading and target staff

(*i*) With the self-reading staff, readings can be taken faster than with the target staff.

(*ii*) In target staff, adjusting the target is often tedius, and staff man must be a relatively trained person capable of coordinating with the instrument man. In the case of a self-reading type, on the other hand, ordinary man can hold the staff concentrating more on keeping the staff in plumb.

(*iii*) The refinement provided by the vernier is often lost when adjusting the target so that it is bisected by the line of sight. Thus the degree of precision finally attained is not significantly different in the two types. However, a target staff is valuable when very long sights are to be taken.

Classification of Direct Differential Levelling

The direct differential levelling can be further classified into the following methods.

Simple levelling

This is the easiest and straightforward type of direct levelling in which only one setting of the instrument is done to determine the difference in level between two points. It is only suitable when the distance involved is within the sighting range of the level. The instrument need not necessarily lie on the line joining the points.

Differential, compound or continuous levelling

When the two points whose difference in elevation is required are too far apart, or when there are intervening obstacles or the difference in elevation is too large, one setting of the instrument will be inadequate. In such cases, the method of simple levelling is used with as many intermediate level stations as required to reach the forward station maintaining a continuity.

Check levelling

It is employed for the purpose of checking of elevations which have already been determined. Generally check levelling is done at the end of each day's work from last station to the starting station for checking the day's work. Instead of returning to the starting station, sometimes, the day's work is checked by connecting the last station to a point of known elevation or a B.M.

Fly levelling

When a B.M. of known elevation is available, a series of levels may be run along a general direction to reach some other point whose level can be determined and fixed as a temporary B.M., if necessary, the method is called fly levelling. Fly levelling is also employed for the determination of approximate levels of different points. In such cases, the distances as long as possible between the level and staff are measured.

The fly levelling is done where rapidity but low precision is required. It is normally used for reconnaissance surveys or for approximate checking of the levels.

Profile or longitudinal levelling

It is used in route surveys such as a highway, railway, canal, sewer line. The elevations of points along the proposed centre line, at known (usually regular) distance apart, are obtained by differential or fly levelling. The object is to determine the profile of the ground in a vertical longitudinal section (*cf*., Sec. 6.4).

Cross-section levelling

This type of direct differential levelling is employed to determine the difference of elevations of the gound surface along the lines perpendicular to the centre line of the proposed route. The cross-section levelling is required to determine the configuration across the alignment of the route. Generally, the profile levelling and cross-section levelling are done simultaneously, to help in estimation of the earthwork involved in a route (*cf.*, Sec. 6.4.10).

Reciprocal levelling

It is the accurate determination of the difference in level between two points by setting up the instrument near each of the points in succession and taking observations of staff readings on both. It is useful when it is not feasible to set up the level midway between the points (*cf.*, Sec. 6.4).

Precise levelling

This is a very accurate method of differential levelling for establishing bench marks or where high precision is required. Refine instruments and procedures enhance the accuracy of the work. It is a specialised form of levelling and is usually conducted by government agencies like Survey of India.

Booking and Reducing Levels

Before discussing the booking and reducing levels, the following terms associated with differential levelling must be understood.

Station

In levelling, a station is that point where the levelling staff is held and not where the level is set up. It is a point whose elevation is to be determined or the point that is to be established at a given elevation.

Height of instrument (H.I.)

For any set up of the level, the elevation of the line of sight is called height of instrument.

Back sight (B.S.)

It is the first reading taken on the staff after setting up the instrument. Initially it is usually made to some form of bench mark or to the point whose elevation has already been determined.

The back sight is usually the first reading taken after setting up the instrument to determine the height of instrument. Thus, the height of instrument is equal to the elevation of the station where staff is held plus the reading on the staff.

Fore sight (F.S.)

It is the reading taken on the staff either held at the last point whose elevation is required or held at the turning point just before shifting the instrument. The fore sight is used to determine the elevation of the staff station. The elevation of the staff station is equal to the height of instrument minus the fore sight.

Change point (C.P.)

A change point or turning point (T.P.) is the point where staff is held and after taking fore sight to determine the elevation of the point where the staff is held, the instrument is shifted and back sight is taken at the same point to determine the new height of instrument. Therefore, at the change point both the back sight and fore sight are taken.

Intermediate station (I.S.)

It is the reading taken on a staff held at a point whose elevation is required, but is not a change point or the last point. Intermediate sites are taken at the intermediate stations.

In Fig. 6.3, S_A is the back sight and S_B is the fore sight. h_A+S_A is the height of instrument. Fig. 6.11 shows the levelling operation between stations *A* to *C*. The elevations of the points *a*, *b*, *c*, and *C* are required to be determined. *A* is a bench mark whose elevation is known. The staff is held at *A* and the level is set up at 1. The first reading taken at *A* is the back sight. Then the staff is held at *a*, *b*, and *B*, the corresponding readings are the intermediate sights on *a* and *b*, and the fore sight on *B*. Then the instrument is shifted to the position 2, and the first reading taken on the staff at *B* is the back sight. As at *B* both the fore sight and back sight readings have been taken, it is the change point. After taking back sight at *B*, the staff is held at *c* and *C*, the corresponding readings being the intermediate sight on *c* and the fore sight on *C*.

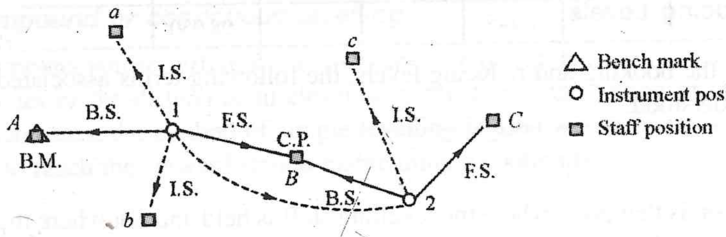


Fig. 6.11 Back sight, fore sight, and intermediate sight

Balancing of sights

When the distances of the points where back sight and fore sight are taken, kept approximately equal, it is known as balancing of sights. This is done to minimise the effect of instrumental and other errors.

Reduced level (R.L.)

Reduced level of a point is the height or depth of the point above or below the assumed datum. It is the elevation of the point. For booking and reducing the levels of points, there are two methods known as (1) Collimation or height of instrument method and (2) Rise and fall method.

Height of instrument method

In this method, height of instrument is calculated for each setting of the instrument by adding the back sight to the known elevation of the point at which back sight has been taken. The reduced level of the points are calculated by subtracting the intermediate sight or fore sight as the case may be, from the height of instrument. The specimen page of a level field book illustrating the method of booking staff readings and reducing the levels of the stations by height of instrument method is shown in Table 6.1.

The station 1 is a bench mark of known elevation of 100.00 m. The staff was held at this station and B.S. 0.685 was taken. Then the staff was held at station 2 and the F.S. 2.905 was taken. Now the instrument was shifted and the B.S. 1.800 was taken at station 2, therefore, this station is a change point. After this two I.S. 1.225 and 2.830 were taken at stations 3 and 4, respectively. The last reading 0.455 for this setting of the instrument is the F.S at station 5, and then the instrument is shifted to another position to take the B.S. 2.550 at station 5. Finally the last reading at station 8 is the F.S. 2.625. Between the stations 5 and 8, an I.S. was taken at station 6, and F.S. and B.S. on station 7.

Table 6.1 Height of instrument method of reduction of levels

Station (a)	B.S. (b)	I.S. (c)	F.S. (d)	H.I. (e)	R.L. (f)	Remarks
1	0.685			100.685	100.000	B.M.
2	1.800		2.905	99.580	97.780	C.P.
3		1.225			98.355	
4		2.830			96.750	
5	2.550		0.455	101.675	99.125	C.P.
6		2.745			98.930	
7	2.590		1.555	102.710	100.120	C.P.
8			2.625		100.085	

For the reduction of levels, the steps are given below:

R.L. 100.000 of St.1 + B.S. 0.685 at St. 1 = H.I. 100.685 in Col. (e)
H.I. 100.685 – F.S. 2.905 at St. 2 = R.L. 97.780 of St. 2 in Col. (f)
R.L. 97.780 of St. 2 + B.S. 1.800 at St. 2 (C.P.) = H.I. 99.580 in Col. (e)
H.I. 99.580 – I.S. 1.225 at St. 3 = R.L. 98.355 of St. 3 in Col. (f)
H.I. 99.580 – I.S. 2.830 at St. 4 = R.L. 96.750 of St. 4 in Col. (f)
H.I. 99.580 – F.S. 0.455 at St. 5 = R.L. 99.125 of St. 5 in Col. (f)
R.L. 99.125 of St. 5 + B.S. 2.550 at St. 5 (C.P.) = H.I. 101.675 in Col. (e)
H.I. 101.675 – I.S. 2.745 at St. 6 = R.L. 98.930 of St. 6 in Col. (f)
H.I. 101.675 – F.S. 1.555 at St. 7 = R.L. 100.120 of St. 7 in Col. (f)
R.L. 100.120 + B.S. 2.590 at St. 7 (C.P.) = H.I. 102.710 in Col. (e)
H.I. 102.710 – F.S. 2.625 at St. 8 = R.L. 100.085 of St. 8 in Col. (f)

The height of instrument method provides the following arithmetic check on the reduction of levels:

$$\Sigma \text{B.S.} - \Sigma \text{F.S.} = \text{Last R.L.} - \text{First R.L.}$$

$$7.625 - 7.540 = 100.085 - 100.000 = 0.085$$

Rise and fall method

In this method, the difference of level between two consecutive points for each setting of the instrument is obtained by comparing their staff readings. The difference indicates a rise if the back sight is more than the fore sight and a fall if it is less than the fore sight. The two consecutive points may have readings as B.S. and F.S, B.S. and I.S., I.S. and I.S., or I.S. and F.S. The rise or fall worked out for all the points, give the difference in level of each point relative to the preceding one. Therefore, if the R.L. of the preceding point is known then the R.L. of the following point can be obtained by adding the rise or subtracting the fall as the case may be.

Table 6.2 shows a page of the level book illustrating the method of calculating the R.L.'s of stations by the rise and fall method. The booking of readings of B.S., I.S. and F.S. is same as in the height of instrument method.

Table 6.2 Rise and fall method of reduction of levels

Station (a)	B.S. (b)	I.S. (c)	F.S. (d)	Rise (e)	Fall (f)	R.L. (g)	Remarks
1	0.685					100.000	B.M.
2	1.800		2.905		2.220	97.780	C.P.
3		1.225		0.575		98.355	
4		2.830			1.605	96.750	
5	2.550		0.455	2.375		99.125	C.P.
6		2.745			0.195	98.930	
7	2.590		1.555	1.190		100.120	C.P.
8			2.625		0.035	100.085	

For the reduction of levels, let us first determine the rise and fall as below:

B.S. 0.685 at St. 1 – F.S. 2.905 at St. 2 = – 2.220 or fall 2.220 in Col. (f)

B.S. 1.800 at St. 2 – I.S. 1.225 at St. 3 = + 0.575 or rise 0.575 in Col. (e)

I.S. 1.225 at St. 3 – I.S. 2.830 at St. 4 = – 1.605 or fall 1.605 in Col. (f)

I.S. 2.830 at St. 4 – F.S. 0.455 at St. 5 = + 2.375 or rise 2.375 in Col. (e)

B.S. 2.550 at St. 5 – I.S. 2.745 at St. 6 = – 0.195 or fall 0.195 in Col. (f)

I.S. 2.745 at St. 6 – F.S. 1.555 at St. 7 = + 1.190 or rise 1.190 in Col. (e)

B.S. 2.590 at St. 7 – F.S. 2.625 at St. 8 = – 0.035 or fall 0.035 in Col. (f)

Now the R.L's of station are calculated as below:

R.L. of St. 1 = 100.000 m (known)

R.L. of St. 2 = R.L. 100.000 of St. 1 – (fall) 2.220 = 97.780 m

R.L. of St. 3 = R.L. 97.780 of St. 2 + (rise) 0.575 = 98.355 m

R.L. of St. 4 = R.L. 98.355 of St. 3 – (fall) 1.605 = 96.750 m

R.L. of St. 5 = R.L. 96.750 of St. 4 + (rise) 2.375 = 99.125 m

R.L. of St. 6 = R.L. 99.125 of St. 5 – (fall) 0.195 = 98.930 m

R.L. of St. 7 = R.L. 98.930 of St. 6 + (rise) 1.190 = 100.120 m

R.L. of St. 8 = R.L. 100.120 of St. 7 – (fall) 0.035 = 100.085 m.

The rise and fall method of reducing the levels, provides two arithmetic checks as below:

$$\Sigma B.S. - \Sigma F.S. = \Sigma Rise - \Sigma Fall = Last\ R.L. - First\ R.L.$$

i.e., $7.625 - 7.540 = 4.140 - 4.055 = 100.085 - 100.000 = 0.085$

Comparision of Height of Instrument Method with Rise and Fall Method

1. The height of instrument method is simple, quicker and less tedius.
2. In the height of instrument method, since the check on the calculations for intermediate sights is not available, the mistakes in their levels pass unnoticed.
3. The rise and fall method though more tedious, provides a full check in the calculations for all sights.
4. The height of instrument method is more suitable in case, where it is required to take a number of readings from the same instrument settings.
5. The rise and fall method should be used for a long run of differential levelling for important works.
6. The rise and fall method should be used when there are a number of important intermediate stations.

ILLUSTRATIVE EXAMPLES

Example 6.1

The following readings were taken with a level and a 4 m staff. Draw up a level book page and reduce the levels by

(a) the rise and fall method.

(b) the height of instrument method.

0.683 B.M.(51.362 m), 1.109, 1.838, 3.399, (3.877 and 0.451) C.P., 1.405, 1.896, 2.676, 3.478, (3.999 and 1.834) C.P., 0.649, 1.706.

Solution:

The first reading is on a B.M. of elevation 51.362 m, and, therefore, it is to be entered as back sight. As the fifth station is a change point 3.877 is to be entered as a fore sight and 0.451 as a back sight. The tenth station is also a change point, therefore, 3.999 is a fore sight and 1.834 is a back sight. The reading 1.706 being on the last station, is a fore sight. All remaining readings are the intermediate sights.

The readings and the reduced levels of the stations are tabulated below:

(a) By rise and fall method

Station	B.S.	I.S.	F.S.	Rise	Fall	R.L.	Remarks
1	0.683					51.362	B.M.
2		1.109			0.426	50.936	
3		1.838			0.729	50.207	
4		3.399			1.561	48.646	
5	0.451		3.877		0.478	48.168	C.P.
6		1.405			0.954	47.214	
7		1.896			0.491	46.723	
8		2.676			0.780	45.943	
9		3.478			0.802	45.141	
10	1.834		3.999		0.521	44.620	C.P.
11		0.649		1.185		45.805	
12			1.706		1.057	44.748	
Σ	2.968		9.582	1.185	7.799		

Arth. Check: 2.968 – 9.582 = 1.185 – 7.799 = 44.748 – 51.362 = – 6.614

(b) By height of instrument method

Station	B.S.	I.S.	F.S.	H.I.	R.L.	Remarks
1	0.683			52.045	51.362	B.M.
2		1.109			50.936	
3		1.838			50.207	
4		3.399			48.646	
5	0.451		3.877	48.168		C.P.
6		1.405			47.214	
7		1.896			46.723	
8		2.676			45.943	
9		3.478			45.141	
10	1.834		3.999	46.454	44.620	C.P.
11		0.649			45.805	
12			1.706		44.748	
Σ	2.968		9.582			

Arth. Check: 2.968 – 9.582 = 44.748 – 51.362 = – 6.614

Example 6.2

It was required to ascertain the elevations of two points *A* and *B* and a line of levels was run from *A* to *B*. The levelling was then continued to bench mark of 77.500 m elevation. The reading obtained being as shown below:

B.S.	I.S.	F.S.	R.L.	Remarks
1.525				*A*
1.776		0.304		
2.005		1.870		
	2.300			*B*
0.899		1.011		
1.913		2.784		
		2.113	77.500	B.M.

Solution:

Since in this problem the starting point *A* is not a station of known elevation, to find the R.L.'s of *A* and *B*, the computation are to be done from bottom to top. To find the height of instrument, the fore sight readings will be added to the R.L. of the known point and to determine the R.L. of the previous point, the back sight will be subtracted from the so obtained H.I. as shown in the table below:

Station	B.S.	I.S.	F.S.	H.I.	R.L.	Remarks
A	1.525			78.989	77.464	*A*
	1.776		0.304	80.461	78.685	
	2.005		1.870		78.591	
B		2.300		80.596	78.296	*B*
	0.899		1.011	80.484	79.585	
	1.913		2.784	79.613	77.700	
			2.113		77.500	B.M.
Σ	8.118		8.082			

Arth. Check : 8.118 – 8.082 = 77.500 – 77.464 = 0.036

Example 6.3

The following consecutive readings were taken with a dumpy level and 5 m levelling staff on continuously sloping ground at a common interval of 15 m. The first point is having an elevation of 185.275 m. Rule out a page of level field book and enter the readings. Calculate (a) the reduced levels of the points by rise and fall method, and (b) the gradient of the line joining the first and the last point.

0.415, 1.025, 2.085, 2.925, 3.620, 4.595, 0.715, 2.115, 3.090, 4.405

Solution:

Since the readings were taken on a continuously sloping ground, the readings are continuously increasing up to sixth reading and then suddenly the seventh reading is lesser than the sixth one showing that the sixth point is a change point. Therefore, the seventh reading is back sight. Again from the seventh reading, the readings are continuously increasing. Thus,

the 1st reading is B.S.,
the 2nd, 3rd, 4th and 5th readings are I.S.'^{s}
the 6th reading is F.S.,
the 7th reading is B.S.,
the 8th and 9th readings are I.S.'^{s,}, and
the 10th reading is F.S.

The page of the level field book with tabulated readings and calculated R.L'^{S} is given below:

(a)

Station	B.S.	I.S.	F.S.	Rise	Fall	R.L.	Remarks
1	0.415					185.275	Known elevation
2		1.025			0.610	184.665	
3		2.085			1.060	183.605	
4		2.925			0.840	182.765	
5		3.620			0.695	182.070	
6	0.715		4.595		0.975	181.095	C.P.
7		2.115			1.400	179.695	
8		3.090			0.975	178.720	
9			4.405		1.315	177.405	
Σ	1.130		9.000	0.000	7.870		

Arth. Check : 1.130 – 9.000 = 0.000 – 7.870 = 177.405 – 185.275 = – 7.870

(b) The gradient of the line

$$= \frac{\text{R.L. of the last point} - \text{R.L. of the first point}}{\text{The length of the line joining the first point to the last point}}$$

$$= \frac{177.405 - 185.275}{15 \times 8}$$

$$= -0.066$$

= **1 in 15.25** (*falling*).

Example 6.4

A page of level field book is reproduced below in which some readings are missing (marked by *). Complete the page with all arithmetical checks.

Station	B.S.	I.S.	F.S.	Rise	Fall	R.L.	Remarks
1	3.65						
2		*		2.75			
3		2.83					
4		3.64					
5	*		7.42				C.P.
6		12.41			7.32		
7		4.32					
8		3.00					
9		– 6.17					Staff held inverted against ceiling
10	*		*			108.26	C.P.
11			*		1.32		
Σ	17.66				25.93		

Solution:

(*i*) B.S. (3.65) at St. 1 – I.S.(*) at St. 2 = 2.75 (rise)

or I.S. at St. 2 = 3.65 – 2.75

= 0.90

(*ii*) B.S.(*) at St. 5 – I.S. (12.41) of St. 6 = 7.32 (fall)

or B.S. at St. 5 = 12.41 + 7.32 (fall)

= 12.41 – 7.32

= 5.09

(*iii*) ΣB.S. = B.S. at St. 1 + B.S. at St.5 + B.S. at St. 10

or 17.66 = 3.65 + 5.09 + B.S. (*) at St. 10

B.S. (*) at St. 10 = 17.66 – 3.65 – 5.09

= 8.92

(*iv*) Now calculate the rise or fall as the case may be, at the stations from 2 to 9 (*cf.*, the tabulated values)

ΣFall = Fall at St. 10 + Σcomputed fall at remaining stations.